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Enhancing Reliability of Series-Parallel systems: A Novel Mathematical Model for Redundancy Allocation

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Abstract. In reliability theory, Series-Parallel configuration systems provide a fundamental framework for examining the relationship between individual component lives and the overall system durability. This research examines the influence of several constraints, namely weight, volume, dimensions, and spatial limitations on enhancing system reliability, specifically regarding spare components for standard oil burners, including nozzle tubes and electrode brackets. An Integrated Redundant Reliability Series-Parallel configuration system is systematically designed and evaluated utilizing the Lagrangean multiplier method, yielding real-valued solutions for essential parameters such as component quantities, component reliability, stage reliability, and overall system reliability. The study used the Dynamic Programming method to seek integer solutions, hence improving the accuracy and relevance of the reliability analysis.

2020 Mathematics Subject Classifications: 90B25, 62N05, 49K30, 90C39, 94C10

Key Words and Phrases: IRR Model, Series-Parallel Configuration, System Reliability, LAM Approach, DMM Approach

1. Introduction

In classical reliability theory, systems and their components are often limited to two distinct states: operational or failed. This binary perspective, however fundamental, constrains the analytical scope by neglecting intermediate situations. The framework of multi-state systems, however, enhances this study by permitting both the entire system and its

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individual components to exist along a continuum of states. This expanded range of options enables a more intricate and comprehensive knowledge of reliability, encompassing the diverse levels of performance and deterioration that arise in real-world situations.

Misra, K. B. [16], created a zero-one integer programming problem in 1971. Lawler and Bell proposed an algorithm to solve the problem. This formulation used arbitrary objective and constraint functions. Mishra studied three optimization problem versions. The formula was simple, and the solving method was easy on a computer. The amount of constraints did not limit the problem's optimal size. Misra, K. B. [17], developed a mathematical framework aimed at improving system reliability under linear constraints. The system was composed of several stages, with each stage utilizing parallel redundancy. The model was converted into a saddle point problem by employing Lagrange multipliers, and Newton's method was employed to resolve the resulting equations, with modifications to enhance computational efficiency. Additionally, the model was extended into a multi-stage decision-making process using the Maximum Principle, offering an efficient, easily implementable solution that ensured convergence and minimized computational effort.

Ei-Neweihi, E. [9], explored the use of Schur-convex functions and majorization techniques to optimize component allocation in parallel-series and series-parallel systems, aiming to improve overall system reliability. In parallel-series systems, the optimal allocation was fully determined by the ranking of component reliabilities, while for series-parallel systems, a partial ordering among allocations was identified to aid in the optimization process. The study further showed that these problems could be reformulated as integer linear programming models, providing exact solutions in some cases, and offering valuable insights in others through the application of Schur function methods. Coit, D.W. [7], introduced a methodology for identifying the optimal design configurations in non-repairable series-parallel systems incorporating cold-standby redundancy. This approach took into account variable component hazard rates and imperfect switching mechanisms. In contrast to previous formulations that presumed perfect switching and exponential time-to-failure, Coit's approach permitted the selection of multiple components per subsystem, with the time-to-failure being modeled by an Erlang distribution. The engineering design problems were more accurately modeled by the methodology, which was effectively demonstrated on a large system with 14 subsystems. This approach, which is based on integer programming, provided significant improvements in the calculation of system reliability.

G. Levitin [13], investigated the influence of system topology modifications on the reliability of units with dual failure modes in 2001. He observed that enhancements in reliability for one failure mode could result in a decrease in reliability for the other. The study suggested an algorithm for optimizing series-parallel topologies in multi-state systems, where the reliability is contingent upon the performance levels of available units. This approach employed a genetic algorithm (GA) for optimization and leveraged the universal moment generating function (UMGF) to quickly assess the reliability of multi-state systems. The paper presented case studies demonstrating the optimization of series-parallel configurations for systems with different performance requirements. In 2004, Jose E. Ramirez-Marquez and David W. Coit [26], developed a redundancy allocation problem (RAP) framework for multi-state series-parallel systems (MSPS) with the goal of reducing design costs while adhering to system-level performance constraints. They addressed the limitations of traditional methods by utilizing capacitated binary components that generate various system performance levels, thereby capturing the multi-state nature of the system. The study introduced a heuristic solution to the MSPS design problem, highlighting its simplicity and effectiveness compared to genetic algorithms (GAs) and its success in improving system reliability.

Y.S. Juang [11], addressed the challenge of system availability in industrial settings, where increasing system complexity leads to higher costs and improved reliability. Traditionally, experienced designers handled the redundancy allocation problem (RAP) in series-parallel systems. To enhance design efficiency, Juang proposed an optimization method using genetic algorithms, which aimed to identify the most cost-efficient strategies for determining components' mean-time-between-failure (MTBF) and mean-time-to-repair (MTTR). Additionally, a knowledge-based interactive decision support system was developed to aid designers in managing component parameters during the design process of repairable series-parallel systems. Similarly, T.C. Chen [6], investigated the nonlinear mixed-integer reliability design problem, focusing on optimizing both the number of redundant components and their reliability within each subsystem to improve overall system performance. Previous studies in this area often employed mathematical programming or heuristic/metaheuristic approaches, but encountered challenges in ensuring feasibility when dealing with nonlinear constraints such as cost, weight, and volume. Chen introduced a penalty-guided artificial immunity algorithm that efficiently explored both feasible and infeasible solution spaces. Numerical examples demonstrated that this method either matched or outperformed the best-known solutions for reliability-redundancy allocation problems.

R.T. Moghaddam [28], presented a genetic algorithm (GA) to tackle the redundancy allocation problem in series-parallel systems. This algorithm enabled the independent selection of redundancy strategies for each subsystem, moving away from the traditional approach where the strategy was fixed and predominantly focused on active redundancy. In real-world systems, however, both active and cold-standby redundancies are often utilized, making the choice of redundancy strategy a key decision variable. The study's computational findings highlighted the GA's robustness and efficiency in addressing the NP-hard problem of determining the optimal redundancy strategy, components, and redundancy level for each subsystem to enhance overall system reliability. Around the same time, Y.C. Liang [14], introduced a variable neighborhood search (VNS) meta-heuristic algorithm to solve the redundancy allocation problem, an NP-hard issue previously constrained by the assumption that each subsystem used identical components. The VNS method overcame this limitation by allowing the simultaneous use of different components, offering a practical approach for large-scale RAP cases. The VNS algorithm was tested on 33 benchmark problems, with results showing its superior performance over the variable neighborhood descent (VND) method, as well as its competitive solution quality when compared to other well-established heuristics like ant colony optimization, genetic algorithms, and tabu search.

Y.S. Jung [11], addressed the challenge of system availability in complex industrial environments, where improving reliability often leads to higher costs. Historically, experienced designers tackled the redundancy allocation problem (RAP) for series-parallel systems. Jung proposed an optimization model based on genetic algorithms to boost design efficiency by determining the most cost-effective strategies for component mean-time-between-failure (MTBF) and mean-time-to-repair (MTTR). In addition, a knowledge-based interactive decision support system was developed to assist designers in managing component parameters throughout the design process of repairable series-parallel systems. J.E.R. Marquez [27], introduced a new formulation of the redundancy allocation problem aimed at maximizing the minimum subsystem reliability in series-parallel systems. This approach provided distinct advantages over traditional formulations by focusing on improving the minimum subsystem reliability, which is critical to the system's overall time-to-failure. Previous reliability designs did not incorporate the combination of functionally equivalent component types within subsystems. However, this method enabled such integration by linearizing the problem and applying integer programming techniques. The methodology's effectiveness in optimizing system reliability was illustrated through three example applications.

M Feizabadi [10], confronted the constraints of conventional reliability optimization models for series-parallel systems in 2015. These models typically necessitated homogeneous components within each subsystem. The study suggested a novel model that facilitated the procurement of components and the design flexibility by allowing for non-homogeneous components. The author devised a genetic algorithm (GA) to address the NP-hard redundancy allocation problem (RAP). The computational results of the GA demonstrated substantial enhancements in system reliability and cost reduction. Rui Peng [24], investigated phased-mission systems (PMSs), which are prevalent in industries such as telecommunications and power transmission and have multiple non-overlapping operational phases. The research concentrated on a particular form of PMS that is composed of subsystems that are connected in series. These subsystems are organized into disjoint work-sharing groups (WSGs) with varying capacities. The minimum capacity of the system's subsystems was used to determine its capacity. He suggested a universal generating function-based approach to evaluate the reliability of capacitated series-parallel PMSs, which takes into consideration imperfect fault coverage. Additionally, he investigated the most effective subsystem structures to optimize reliability. The method and optimization process were illustrated through the use of numerous examples.

N Alikar [3], developed a mixed-integer binary non-linear programming model to solve a novel series-parallel multi-component, multi-periodic inventory-redundancy allocation problem (IRAP). The IRAP implemented an all-unit discount policy for component purchases, in addition to imposing restrictions on budget, storage, vehicle capacities, and system weight. The objective was to minimize total costs, which included ordering, holding, and purchasing costs, while maximizing system reliability. To address infeasible solutions, a penalty function was implemented. He employed the NSGA-II, MOPSO, and MOHS algorithms to achieve optimal Pareto solutions. The numerical examples for the proposed IRAP favored NSGA-II. Pourkarim Guilani et al. [25], expanded the redundancy allocation problem (RAP) by proposing a bi-objective model (BORAP) that incorporates components with three states, as opposed to the conventional binary states of "fully operational" or "entirely failed." The research simulated a system comprising serial subsystems, whereby non-repairable tri-state components were arranged in parallel according to the k-out-of-n policy, and it included technological and organizational elements influencing failure rates. The aim was to enhance system reliability while reducing costs, achieved by the multi-objective strength Pareto evolutionary algorithm (SPEA-II), with validation conducted using the non-dominated sorting genetic algorithm (NSGA-II) over 20 test cases.

Sridhar Akiri et al. [2], performed an extensive investigation into the design, analysis, and optimization of an integrated coherent redundant reliability system, a topic that had not been previously documented. The system's architecture was initially evaluated with the Lagrangean multiplier, and integer solutions were obtained to enhance reliability through integer and dynamic programming methods, ensuring practical applicability. S A Abed et al. [1], investigated optimal reliability methodologies for allocating reliability values in series-parallel systems to save money. Generalized cost formulations were used in the nonlinear programming problem. A "series-parallel system" was changed into an equivalent "series system", examined with uniform component reliability, constructed utilizing parallels with other engineering issues, and emphasized dependability system-electrical circuit duality. Srinivasa Rao Velampudi et al. [30], performed an investigation including extraneous reliability into structured systems by the application of Lagrangian multipliers and dynamic programming techniques. A heuristic method was utilized to produce an integer solution, enhancing system efficiency by evaluating aspects including cost, size, and load. The proposed method sought to improve system performance by assessing phase reliabilities and factor efficiencies, with outcomes demonstrated using a numerical example.

Jing Liao et al. [15], introduced a Reliability Allocation-based Programming Model (RAPM) to enhance maintenance techniques for series-parallel systems, tackling the varied deterioration and failure patterns of system components. The model assigned component dependability according to weights and implemented stringent safety requirements to improve system availability, reduce maintenance costs, and diminish reliability gaps. The RAPM offered insights into patterns in component states, exemplified by a case study on traction converter systems in electric locomotives. The model, validated with actual data, forecasted repair schedules for planned downtimes, presenting possible economic advantages for railway firms.

Srinivasa Rao Velampudi et al. [29], performed a case study on the Muffle Box Furnace to enhance system efficiency. The study employed Lagrangean methods to compute the price, weight, and volume components for several system configurations, resulting in the creation of a United Reliability Model (URM). The research incorporated value constraints into IRR Models, highlighting the correlation between component cost and dependability, while also integrating weight and volume as supplementary limitations.

Bhavani Kapu et.al. [12], introduced the Integrated Redundant Reliability Model (IRRM), which improves system reliability via a parallel-series arrangement. The model, intended for critical systems, employed Lagrangian techniques and adaptations of the

Newton-Raphson method to enhance component efficiency, phase reliability, and overall system performance, offering support for single-phase AC synchronous generators and guaranteeing operational continuity despite subsystem failures.

The present manuscript builds upon a review of existing literature in the domain of reliability theory and non linear constraints [4, 5, 8, 18–23, 31]. The following studies were thoroughly examined to provide a foundation for the research.

In the present paper, the authors performed a comprehensive study on Series-Parallel configurations in relation to Integrated Redundant Reliability (IRR) Models, emphasizing redundant reliability arrangements. This research utilized a comprehensive case study focused on spare parts frequently utilized in oil burners, including strainers, nozzle tubes, and electrode brackets. The study provided substantial insights into design concerns and the development of integrated reliability systems, ultimately advancing both engineering practice and the field of reliability theory.

This study investigated a series-parallel configuration by developing an IRR and employing the conventional Lagrange multipliers method to get real-valued solutions, considering both rounded and unrounded outcomes. The "Dynamic Programming" method was introduced as an innovative technique for generating integer values, enabling a comparison with the Lagrangean method and providing scientifically rigorous solutions. This methodology sought to preserve the necessary quantity of components ($t_{\beta}j$) at each phase while improving overall system dependability (R_{StRe}).

2. Some Definitions and Notations

Reliability analysis plays a crucial role in assessing the performance of systems and their components under specified conditions. Uniformity is assumed among elements within each stage, signifying that all elements share an equivalent level of reliability. Statistical independence is attributed to all elements, implying that the failure of one element exerts no influence on the functionality of other elements within the structure. The following definitions provide clarity on key concepts such as component, stage, system reliability, and the Integrated Redundant Reliability (IRR) Model,

Definition 2.1. Component reliability is the likelihood that a system component will perform its intended function without failure under specified operating conditions and timeframes. It indicates the component's dependability to contribute to system functionality and efficiency while considering design, material quality, and environmental circumstances.

Definition 2.2. Stage reliability is the likelihood that an entire stage within a system, comprising multiple interconnected components, will operate successfully without failure under specified conditions for a given period. It accounts for the combined performance of all components in the stage, considering their configuration and interactions, to ensure the stage fulfills its intended function within the system.

Definition 2.3. System reliability refers to the probability that an entire system will perform its intended function without failure for a specified duration under defined operating conditions. It reflects the collective performance of all components and stages within the system, accounting for their configuration and interdependence to ensure overall functionality and dependability.

Definition 2.4. The Integrated Redundant Reliability (IRR) Model evaluates redundancy-based systems for continuous operation. It examines how primary and redundant components work together to prevent failure. The model considers how primary and redundant components interact to improve system performance and reduce downtime to determine system dependability.

R_{StRe} = Systems Efficiency in Series-Parallel Configuration
$R_{\beta pj}$ = Process Phase Efficiency 'βj ', $0 < R_{\beta pj} < 1$
$r_{\beta j}$ = Component Efficiency in the Phase 'j'; Where $0 < r_{\beta j} < 1$
$t_{\beta j}$ = Number of items in Phase 'βj '
C_{pc} = Item's-Price factor for each element in the phase 'βj '
W_{wc} = Item's-Weight factor for each element in the phase 'βj '
V_{vc} =Item's-Volume factor for each element in the phase 'βj '
$C_{\beta 0}$ = Maximum permissible system – Component's-Price
$W_{\beta 0}$ = Maximum permissible system - Component's-Weight
$V_{\beta 0}$ = Maximum permissible system - Component's-Volume
LMT Lagrange Multiplier Technique
DPT Dynamic Programming Technique
IRRM Integrated Reliability and Redundancy Model
$b_\beta, f_\beta, i_\beta, d_\beta, k_\beta, n_\beta$ are Constants.

3. The Model

The equations were obtained using Lagrange's method of undetermined multipliers. This method can also be utilized by students from non-engineering backgrounds and researchers to determine the extremum values of a function $f(x, y, z)$, subject to the condition $\phi(x, y, z) = 0$.

The Lagrangian function is expressed as: $F(x, y, z) = f(x, y, z) + \lambda\phi(x, y, z)$, where λ is called Lagrangean multiplier.

Note 3.1. To find the extremum values for a function $f(x, y, z)$,subject to the conditions $\phi(x, y, z) = 0 \& Phi(x, y, z) = 0$, the Lagrangian function is formulated as:$F(x, y, z) = f(x, y, z) + \lambda\phi(x, y, z) + \mu\lambda(x, y, z)$,, where λ and μ are the Lagrange multipliers.

The system's dependability concerning the given value function
Maximize

$$R_{StRe} = 1 - \Pi_{\beta=1}^{k}\Big[1 - \Pi_{j=1}^{n} R_{\beta pj}\Big] \tag{1}$$

The subsequent correlation between value and efficiency is employed to determine the value coefficient of each unit in the phase βj.

$$r_{\beta j} = \cot^{-1} h\big[\frac{c_j}{b_j}\big]^{1/d_j} \tag{2}$$

Therefore,

$$C_{pc} = b_j \coth{(r_{\beta j})}^{d_j} \tag{3}$$

Similarly,

$$W_{wc} = f_j \coth{(r_{\beta j})}^{k_j} \tag{4}$$

$$V_{vc} = i_j \coth{(r_{\beta j})}^{n_j} \tag{5}$$

Since component's-price is linear in 'βj',

$$\sum_{j=1}^{n} C_{pc} t_{\beta j} \leq c_0 \tag{6}$$

Similarly component's-weight and component's-volume are also linear in 'βj'.

$$\sum_{j=1}^{n} W_{wc} t_{\beta j} \leq w_0 \tag{7}$$

$$\sum_{j=1}^{n} V_{vc} t_{\beta j} \leq v_0 \tag{8}$$

From (3), (4), (5) we get

$$\sum_{j=1}^{n} b_j \coth{(r_{\beta j})}^{d_j} t_{\beta j} - C_{\beta 0} \leq 0 \tag{9}$$

$$\sum_{j=1}^{n} f_j \coth{(r_{\beta j})}^{k_j} t_{\beta j} - W_{\beta 0} \leq 0 \tag{10}$$

$$\sum_{j=1}^{n} i_j \coth{(r_{\beta j})}^{n_j} t_{\beta j} - V_{\beta 0} \leq 0 \tag{11}$$

The transformed equation through the relation

$$t_{\beta j} = \frac{\log R_{\beta p j}}{\log r_{\beta j}} \tag{12}$$

Where, $R_{StRe} = \Pi_{\alpha=1}^{k}[1-(1-r_{\beta j})^{t_{\beta j}}]$
Subject to the constraints

$$\sum_{j=1}^{n} b_j \coth{(r_{\beta j})}^{d_j} \frac{\log R_{\beta p j}}{\log r_{\beta j}} - C_{\beta 0} \leq 0 \tag{13}$$

$$\sum_{j=1}^{n} f_j \coth{(r_{\beta j})}^{k_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - W_{\beta 0} \leq 0 \tag{14}$$

$$\sum_{j=1}^{n} i_j \coth{(r_{\beta j})}^{n_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - V_{\beta 0} \leq 0 \tag{15}$$

Non-negative restrictions $\beta j \geq 0$
A Lagrangean function is defined as

$$F = R_{\beta pj} + \lambda_0 \left[\sum_{j=1}^{n} b_j \coth{(r_{\beta j})}^{d_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - C_{\beta 0} \right] + \beta_0 \left[\sum_{j=1}^{n} f_j \coth{(r_{\beta j})}^{k_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - W_{\beta 0} \right] + \delta_0 \left[\sum_{j=1}^{n} i_j \coth{(r_{\beta j})}^{n_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - V_{\beta 0} \right] \tag{16}$$

Utilizing the Lagrangean function enables the identification of the optimal point and its separation by where $R_{\beta pj}, r_{\beta j}, \lambda_0, \beta_0, \delta_0$ are idle points.

$$\frac{\partial F}{\partial R_{\beta pj}} = 1 + \lambda_0 \left[\sum_{j=1}^{n} b_j \coth{(r_{\beta j})}^{d_j} \frac{1}{R_{\beta pj} \log r_{\beta j}} \right] + \beta_0 \left[\sum_{j=1}^{n} f_j \coth{(r_{\beta j})}^{k_j} \frac{1}{R_{\beta pj} \log r_{\beta j}} \right] + \delta_0 \left[\sum_{j=1}^{n} i_j \coth{(r_{\beta j})}^{n_j} \frac{1}{R_{\beta pj} \log r_{\beta j}} \right] \tag{17}$$

$$\frac{\partial F}{\partial r_{\beta j}} = \sum_{j=1}^{n} b_j \coth{(r_{\beta j})}^{d_j - 1} \frac{\log R_{\beta pj}}{\log r_{\beta j}} \left(- d_j cosech^2(r_\beta j) - \frac{\coth{(r_{\beta j})}}{r_{\beta j} (\log r_{\beta j})} \right) + \sum_{j=1}^{n} f_j \coth{(r_{\beta j})}^{k_j - 1} \frac{\log R_{\beta pj}}{\log r_{\beta j}} \left(- k_j cosech^2(r_\beta j) - \frac{\coth{(r_{\beta j})}}{r_{\beta j} (\log r_{\beta j})} \right) + \sum_{j=1}^{n} i_j \coth{(r_{\beta j})}^{n_j - 1} \frac{\log R_{\beta pj}}{\log r_{\beta j}} \left(- n_j cosech^2(r_\beta j) - \frac{\coth{(r_{\beta j})}}{r_{\beta j} (\log r_{\beta j})} \right) \tag{18}$$

$$\frac{\partial F}{\partial \lambda_0} = \sum_{j=1}^{n} b_j \coth{(r_{\beta j})}^{d_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - C_{\beta 0} \tag{19}$$

$$\frac{\partial F}{\partial \beta_0} = \sum_{j=1}^{n} f_j \coth{(r_{\beta j})}^{k_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - W_{\beta 0} \tag{20}$$

$$\frac{\partial F}{\partial \delta_0} = \sum_{j=1}^{n} i_j \coth{(r_{\beta j})}^{n_j} \frac{\log R_{\beta pj}}{\log r_{\beta j}} - V_{\beta 0} \tag{21}$$

where, λ_0, β_0 and δ_0 are Lagrangean multipliers.

Utilizing the Lagrangean approach, we ascertain the quantity of elements in each phase ($t_{\beta j}$), identify the optimal reliability of components ($r_{\beta j}$), calculate the reliability of each stage ($R_{\beta pj}$), and assess the overall structural reliability (R_{StRe}). This method provides a definitive numerical solution concerning the component's cost, weight, and volume.

Mechanical system performance, safety, cost-efficiency, and lifespan depend on component reliability. In aerospace and automotive, reliability is crucial to prevent breakdowns because a system's weakest component determines its reliability. Reliable parts save money, last longer, and satisfy customers. Exponential and Weibull Distributions, MTTF, and environmental factors are used to calculate component reliability, which is the chance of a component working as intended without failure under particular conditions for a set duration. This work presents an integrated redundant reliability model that calculates component and stage reliabilities using the Lagrangean multiplier approach and dynamic programming.

4. Case Study

This study utilizes optimization methods to ascertain parameters for a particular mechanical system, based on the premise that the cost, weight, and volume factors are directly proportional to efficiency of system. This assumption may not be applicable to electronic systems. Thus, the evaluation of maximal component level reliability ($r_{\beta j}$), stage reliability ($R_{\beta pj}$), amount of elements per stage ($t_{\beta j}$), and structural accuracy ($R_{\beta pj}$) is pertinent to any mechanical system. A comprehensive overview of research on the optimal assignment of components in series-parallel systems reveals valuable insights into exact algorithms, redundancy allocations and interchangeable components approaches. These studies contribute to enhancing our understanding of reliability engineering in complex systems. This study specifically focuses on evaluating the structure accuracy of a specialized machine designed for the assembly of Typical Oil Burner. The schematic diagram of the Typical Oil Burner is shown in Figure 1.

An oil burner is a heating apparatus that utilizes oil as fuel to produce heat, frequently used in home, commercial, and industrial heating systems. The process involves atomizing the oil into minute droplets, which are subsequently combined with air and ignited, generating a regulated flame that heats air or water in boilers, furnaces, or water heaters. Oil burners are prevalent in colder regions where oil is a more economical or available fuel alternative to gas or electricity. Their efficiency and comparatively clean combustion render them a dependable option for heating requirements. They are generally employed in structures lacking access to natural gas pipes or as contingency systems during gas supply interruptions. The essential elements of a standard oil burner comprise the fuel pump, which conveys oil to the nozzle at the appropriate pressure; the nozzle, which atomizes the oil; an ignition transformer, which generates the requisite spark to ignite the oil-air mixture; and the air blower, which furnishes the necessary air for combustion. Additional spare components including the burner motor, electrodes, fuel filter, and control box. The cost of oil burners typically ranges from $500 to $1,500 USD, influenced on their capacity

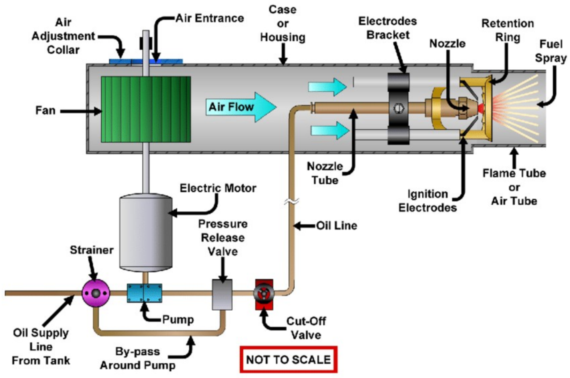

Figure 1: Typical Oil Burner

and features. Weights often vary from 40 to 80 pounds, although the volume of a normal oil burner may range from 16.66 to 49.98 cubic meters, contingent upon the brand and kind.

4.1 Parameters of the Case Problem

The constants necessary for the case problem are presented in Table 1.

Table 1: Preset Constant Values for Parameters of Value, Load, and Size in Series-Parallel Configuration Systems

Phase	Constituents of Value		Constituents of Load		Constituents of Dimension	
	b_j	d_j	f_j	k_j	i_j	n_j
I	1000	0.91	40	0.92	16.66	0.94
II	1200	0.92	60	0.94	33.33	0.89
III	1500	0.93	80	0.96	49.98	0.86

The Table 2, Table 3 and Table 4 below display the structural efficiency, as well as the efficiency of each factor, phase, and number of factors in each stage.

4.2 Analysis of Component Worth Constraints Using LMT

Figure 2 illustrates the component efficiencies ($r_{\beta j}$) and process phase efficiencies ($R_{\beta pj}$) of price constraint obtained through approximately 50 iterations of a trial-and-error method using a MATLAB program. The program was developed to construct an integrated redundant reliability model based on a series-parallel configuration, taking into account constraints such as cost, weight, and size.

4.3 Analysis of Component Load Constraints Using LMT

Here in Table 2 delineates the value-related efficiency design, the authors selected the optimal component efficiencies ($r_{\beta j}$) and process phase efficiencies ($R_{\beta pj}$) based on price

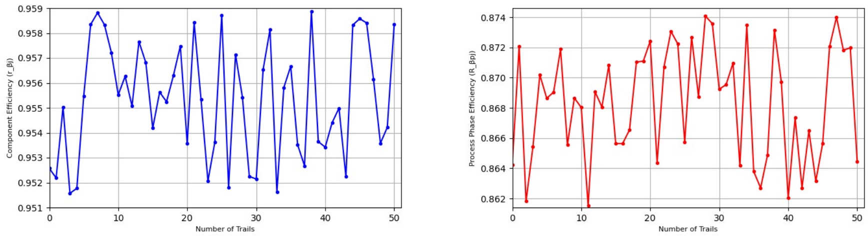

Figure 2: The Component $(r_{\beta j})$ & the Process Phase Efficiencies $(R_{\beta pj})$ of Price Components by Using LMT

constraints from these iterations. Using these optimal values, the number of components required for each phase was determined, along with the corresponding total cost and overall system efficiency.

Table 2: Analysis of Price Constraints in Series-Parallel Configuration Systems Using the LMT

Phase	b_j	d_j	$r_{\beta j}$	$\log r_{\beta j}$	$R_{\beta pj}$	$\log R_{\beta pj}$	$t_{\beta cj}$	C_{pc}	$C_{pc} \cdot t_{\beta cj}$
I	1000	0.91	0.9589	-0.0182	0.8625	-0.0642	3.52	1118.19	3935.36
II	1200	0.92	0.9514	-0.0216	0.8745	-0.0582	2.69	1328.67	3575.01
III	1500	0.93	0.9523	-0.0212	0.8615	-0.0647	3.05	1661.66	5069.11
	Ultimate Value								12579.48
	Efficiency of System$(R_{\beta pj})$								0.9684

4.4 Analysis of Component Size Constraints Using LMT

Figure 3 illustrates the component efficiencies $(r_{\beta j})$ and process phase efficiencies $(R_{\beta pj})$ of load constraint obtained through approximately 50 iterations of a trial-and-error method using a MATLAB program. The program was developed to construct an integrated redundant reliability model based on a series-parallel configuration, taking into account constraints such as cost, weight, and size. Here in Table 3 delineates the load-

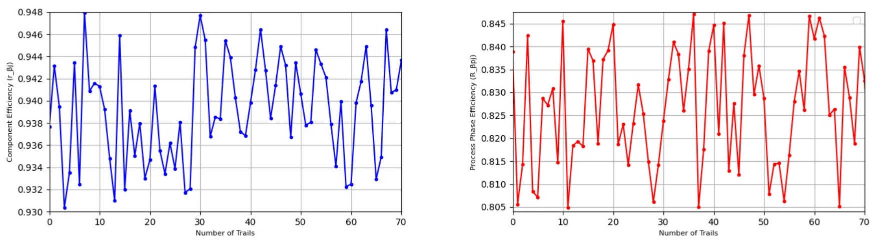

Figure 3: The Component $(r_{\beta j})$ & the Process Phase Efficiencies $(R_{\beta pj})$ of Load Components by Using LMT

related efficiency design, the authors selected the optimal component efficiencies $(r_{\beta j})$ and process phase efficiencies $(R_{\beta pj})$ based on price constraints from these iterations. Using these optimal values, the number of components required for each phase was determined, along with the corresponding total load and overall system efficiency.

Table 3: Analysis of Load Constraints in Series-Parallel Configuration Systems Using the LMT

Phase	f_j	k_j	$r_{\beta j}$	$\log r_{\beta j}$	$R_{\beta pj}$	$\log R_{\beta pj}$	$t_{\beta wj}$	W_{wc}	$W_{wc} \cdot t_{\beta wj}$
I	40	0.92	0.9475	-0.0234	0.8471	-0.0721	3.08	54	166.32
II	60	0.94	0.9304	-0.0313	0.8347	-0.0785	2.50	82	205.01
III	80	0.96	0.9414	-0.0262	0.8047	-0.0944	3.60	109	392.41
	Ultimate Load								763.74
	Efficiency of System($R_{\beta pj}$)								0.9684

Figure 4 illustrates the component efficiencies ($r_{\beta j}$) and process phase efficiencies ($R_{\beta pj}$) of size constraint obtained through approximately 50 iterations of a trial-and-error method using a MATLAB program. The program was developed to construct an integrated redundant reliability model based on a series-parallel configuration, taking into account constraints such as cost, weight, and size. Here in Table 4 delineates the load-

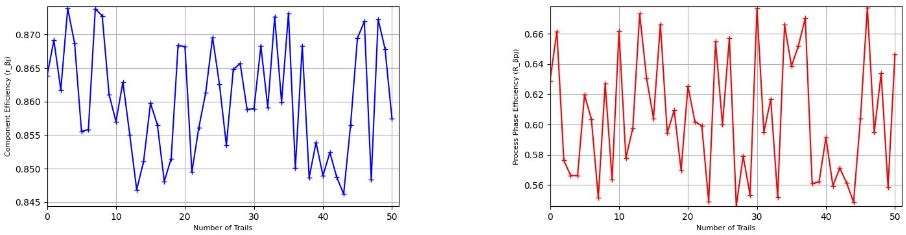

Figure 4: The Component ($r_{\beta j}$) & the Process Phase Efficiencies ($R_{\beta pj}$) of Size Components by Using LMT

related efficiency design, the authors selected the optimal component efficiencies ($r_{\beta j}$) and process phase efficiencies ($R_{\beta pj}$) based on size constraints from these iterations. Using these optimal values, the number of components required for each phase was determined, along with the corresponding total load and overall system efficiency.

Table 4: Analysis of Size Constraints in Series-Parallel Configuration Systems Using the LMT

Phase	i_j	n_j	$r_{\beta j}$	$\log r_{\beta j}$	$R_{\beta pj}$	$\log R_{\beta pj}$	$t_{\beta vj}$	V_{vc}	$V_{vc} \cdot t_{\beta vj}$
I	16.66	0.94	0.8741	-0.0584	0.6777	-0.1690	2.89	17	49.13
II	33.33	0.89	0.8445	-0.0734	0.6487	-0.1880	2.56	32	81.92
III	49.98	0.86	0.8456	-0.0728	0.5461	-0.2627	3.61	49	176.89
	Ultimate Dimension								307.94
	Efficiency of System($R_{\beta pj}$)								0.9684

5. Optimization of efficiency through the application of the Lagrange Multiplier method

The efficiency of system consolidates the 'βj' values as integers by rounding 'βj' to the nearest whole number, while the permissible outcomes for worth, load, and size are enumerated in the tables. Compute the variation attributable to value, load, dimensions, and construction capacity (both prior to and subsequent to including rounding-off 'βj' to the nearest integer) to acquire data.

5.1 Efficiency Design Using LMT for Value, Load, and Size with Rounding-Off

Table 5: Analysis of Efficiency Design Concerning Value, Load, and Size Constraints Utilizing the Lagrange Multiplier Method Including Rounding-off Techniques, Presented in the Following Table

Phase	$r_{\beta j}$	$R_{\beta pj}$	$t_{\beta cj}$	C_{pc}	$C_{pc} \cdot t_{\beta cj}$	$t_{\beta wj}$	W_{wc}	$t_{\beta j} \cdot W_{wc}$	$t_{\beta vj}$	V_{vc}	$t_{\beta wj} \cdot V_{vc}$
I	0.9589	0.8625	3	1342	4026	4	54	216	4	17	68
II	0.9514	0.8745	3	1617	4851	3	82	246	3	32	96
III	0.9523	0.8602	4	2021	8084	3	109	327	3	49	147
Total Worth, Load and Size				16961		789			311		
Efficiency of System($R_{\beta pj}$)									0.9763		

5.1.1

Price Fluctuation $= \frac{\text{(Rounded Total Price - Exact Total Price)}}{\text{(Exact Total Price)}} = 34.83\%$

5.1.2

Weight Fluctuation $= \frac{\text{(Rounded Total Weight - Exact Total Weight)}}{\text{(Exact Total Weight)}} = 03.03\%$

5.1.3

Volume Fluctuation $= \frac{\text{(Rounded Total Volume - Exact Total Volume)}}{\text{(Exact Total Volume)}} = 11.38\%$

5.1.4

Fluctuation in Efficiency $= \frac{\text{(Rounded Total Efficiency - Exact Total Efficiency)}}{\text{(Exact Total Efficiency)}} = 01.00\%.$

6. Dynamic Programming Technique

The Lagrangean technique has some limitations, including the necessity to specify the quantity of components required at each stage (βj) in actual values, which might be challenging to implement. The prevalent method of truncating result values alters worth, load, and size, so influencing system reliability and substantially impacting the model's efficiency design. The author proposes an alternative empirical implementation to address this problem, utilizing the dynamic programming method to derive an integer solution, employing the solutions generated by the Lagrangian approach as parameters for the dynamic programming method.

6.1 Optimizing System Reliability and Resource Constraints Using Dynamic Programming in Series-Parallel Configuration

In the context of series-parallel configurations using an integrated redundant model, dynamic programming can be employed to address optimization problems related to cost, load, size, and system reliability. By leveraging Python programming, the approach involves decomposing the primary problem into smaller sub-problems and systematically storing their solutions to eliminate redundant computations. This method is particularly effective when dealing with problems that exhibit overlapping substructures and have optimal sub-problem solutions. The following outlines a structured approach for dynamic programming:

1. **Problem Definition**: Clearly specify the problem to be addressed, including the objective to be optimized. Identify the relevant parameters, variables, and constraints, such as cost, load, and size in the integrated redundant model.

2. **Identify Optimal Substructure**: Decompose the problem into smaller sub-problems that mirror the structure of the original. These sub-problems should contribute to solving the overall system, including reliability and performance within the constraints of price and load.

3. **Formulate Recurrence Relations**: Establish recurrence relations that express the solution of each sub-problem in terms of the solutions to its smaller sub-problems. This creates a relationship between sub-problems, allowing for systematic solution building.

4. **Construct a Memorization Table**: To avoid repeated calculations, create a table or array that stores the solutions to subproblems as they are solved. This technique, known as memorization, starts with base cases that represent the simplest sub-problem solutions.

5. **Populate the Table**: Use the recurrence relations to fill in the table, starting from the smallest sub-problems and working upward. By reusing solutions from previously solved sub-problems, you efficiently build the solution to the larger problem.

6. **Retrieve the Final Solution**: Once the table is fully populated, the final solution to the original problem can be found by referencing the relevant entry in the table, providing insight into overall system reliability and optimization.

7. **Space Optimization (Optional)**: In some cases, memory usage can be optimized by only storing essential values from the table, particularly if only a subset of the solutions is required for further analysis.

8. **Bottom-Up Approach (Optional)**: Dynamic programming can be implemented either in a bottom-up or top-down manner. A bottom-up approach solves the smallest sub-problems first, iteratively building up to the complete solution, while a top-down approach employs recursion combined with memorization.

9. **Analyze Complexity**: Evaluate the time and space complexity of the dynamic programming solution. This analysis often reveals significant reductions in computational effort compared to more straightforward brute-force methods.

10. **Test and Validate**: Thoroughly test the dynamic programming solution using various inputs to ensure correctness. Verify that the results stored in the table align with the expected outcomes.

Dynamic programming is an effective technique applicable to a wide variety of optimization problems, including those related to reliability models, resource allocation, and system performance. Practicing this approach is essential for recognizing problems that can benefit from dynamic programming and for developing efficient, reliable solutions in Table 6, Table 7 and Table 8 in the below.

Table 6: Initial Stage of the Dynamic Programming Process

Phase-I(βj)	Phase-I-Reliability ($R_{\beta pj}$)
01	0.7524
02	0.8154
03	0.8542
04	0.8647
05	**0.9224**

Table 7: Dynamic Programming, Second Stage

Phase-II (βj)	Phase-II-Reliability ($\mathbf{R}_{\beta pj}$)							
06	0.6874	0.7916	0.7978	0.9238				
07	0.6962	0.7342	0.7416	0.9125	0.8991			
08	0.7258	0.8768	0.7854	0.9012	0.8496	0.9285		
09	0.7406	0.7981	0.8073	0.8899	0.9571	0.9514	0.8934	0.8523
10	0.7554	0.8194	0.8292	0.8856	0.9224	0.9258	0.8954	0.8835
11	0.7728	0.8407	0.8511	**0.9571**	0.9125	0.9144	0.9247	0.9264

Table 8: Dynamic Programming, Final Stage

Phase-III (βj)	Phase-III-Reliability ($\mathbf{R}_{\beta pj}$)							
04	0.8232	0.8235	0.9341					
05	0.8383	0.8045	0.9144	0.9451				
06	0.8824	0.8436	0.9454	0.9421	0.9345			
07	0.8265	0.8874	0.9142	0.9514	0.9341	0.8999		
08	0.8706	0.8951	0.9711	0.9354	0.9047	0.9243	0.9331	
09	0.8147	0.9356	0.9573	0.9125	**0.9945**	0.9494	0.9222	0.9354

7. Results

The Lagrange multiplier method provided a continuous solution for the proposed Integrated Redundant Reliability (IRR) Systems, which were modeled using series-parallel configurations. These configurations, commonly used in reliability engineering, combine components in both series and parallel arrangements to enhance system reliability. In the models under investigation, the method not only offered a real-valued (continuous) solution but also worked alongside the necessary integer solution required for practical application.

To further clarify and interpret the most critical findings of these models, the Dynamic Programming Approach was applied. This approach helped optimize decisions at each stage of the process, ensuring that both the series and parallel components of the IRR system were effectively managed to maximize reliability. The results of the mathematical function, which evaluated the performance and reliability of these systems, are summarized and presented in detail in Table 9, Table 10, and Table 11. These tables illustrate how the combined use of Lagrange multipliers and dynamic programming contributed to solving the complex reliability problem posed by the series-parallel configuration in the IRR model.

7.1 Dynamic Programming Analysis of Component Worth Constraints

Detailed information regarding the worth-related efficiency design can be found in Table 9.

Table 9: Detailed Analysis of Component Price Constraints Utilizing a Dynamic Programming Approach in Series-Parallel Configuration Systems

Phase	f_j	d_j	$r_{\beta j}$	$\log r_{ej}$	$R_{\beta pj}$	$\log R_{sj}$	$t_{\beta cj}$	C_{pc}	$t_{\beta cj} \cdot C_{pc}$
I	1000	0.91	0.9622	-0.0167	0.8571	-0.0670	4	1120	4482
II	1200	0.92	0.9601	-0.0177	0.8849	-0.0531	3	1342	4025
III	1500	0.93	0.9734	-0.0117	0.9224	-0.0351	3	1704	5113
Ultimate Worth									13620

Variation in the Worth-Component = 19.69%
Variation in Change of Efficiency in System = 01.00%.

7.2 Dynamic Programming Analysis of Component Load Constraints

Detailed information regarding the load-related efficiency design can be found in Table 10.

Table 10: Detailed Analysis of Component Load Constraints Utilizing a Dynamic Programming Approach in Series-Parallel Configuration Systems

Phase	p_j	k_j	$r_{\beta j}$	$\log r_{ej}$	$R_{\beta pj}$	$\log R_{sj}$	$t_{\beta wj}$	W_{wc}	$t_{\beta wj} \cdot W_{wc}$
I	40	0.92	0.9706	-0.0130	0.9145	-0.0388	3	53	159
II	60	0.94	0.9736	-0.0116	0.9229	-0.0348	3	80	240
III	80	0.96	0.9891	-0.0048	0.9571	-0.0190	4	106	424
Ultimate Load									823

Variation in the Load-Component = 04.031%
Variation in Change of Efficiency in System = 01.23%.

7.3 Dynamic Programming Analysis of Component Size Constraints

Detailed information regarding the size-related efficiency design can be found in Table 11.

Table 11: Detailed Analysis of Component Size Constraints Utilizing a Dynamic Programming Approach in Series-Parallel Configuration Systems

Phase	q_j	n_j	$r_{\beta j}$	$\log r_{ej}$	$R_{\beta pj}$	$\log R_{sj}$	$t_{\beta vj}$	V_{vc}	$t_{\beta vj} \cdot V_{vc}$
I	16.66	0.94	0.9982	-0.0008	0.9945	-0.0024	3	20	60
II	33.33	0.89	0.9736	-0.0116	0.9229	-0.0348	3	38	114
III	49.98	0.86	0.9891	-0.0048	0.9571	-0.0190	4	58	232
Ultimate Dimension									406
Efficiency of System($R_{\beta pj}$)									0.9901

Variation in the Volume-Component = 03.55%
Variation in Change of Efficiency in System = 01.23%.

7.4 Comparative Analysis of Optimization Methods for Worth Using the IRRM: LMT vs. DPT

Table 12: Results Correlating the Lagrange Multiplier Method Including a Rounding-off Technique and Dynamic Programming Approaches for Pricing in Series-Parallel Configuration Systems

		Including a Rounding Off				Dynamic Programming			
Phase	$t_{\beta j}$	$r_{\beta j}$	$R_{\beta pj}$	C_{pc}	$t_{\beta cj} \cdot C_{pc}$	$r_{\beta j}$	$R_{\beta pj}$	C_{pc}	$t_{\beta cj} \cdot C_{pc}$
I	3	0.9589	0.8625	1342	4026	0.9622	0.8571	1120	4482
II	3	0.9514	0.8745	1617	4851	0.9601	0.8849	1342	4025
III	4	0.9523	0.8602	2021	8084	0.9734	0.9224	1704	5113
Ultimate Value		16961				13620			
Efficiency of System($R_{\beta pj}$)		Applying LMT($R_{\beta pj}$)			0.9763	Applying DPT($R_{\beta pj}$)			0.9891

7.5 Comparative Analysis of Optimization Methods for Load Using the IRRM: LMT vs. DPT

Table 13: Results Correlating the Lagrange Multiplier Method with Including a Rounding-off Technique and Dynamic Programming Approaches for Loading Series-Parallel Configuration Systems

		Including a Rounding Off				Dynamic Programming			
Phase	$t_{\beta j}$	$r_{\beta j}$	$R_{\beta pj}$	W_{wc}	$t_{\beta wj} \cdot W_{wc}$	$r_{\beta j}$	$R_{\beta pj}$	W_{wc}	$t_{\beta wj} \cdot W_{wc}$
I	4	0.9589	0.8625	54	216	0.9706	0.9145	53	159
II	3	0.9514	0.8745	82	246	0.9736	0.9229	80	240
III	3	0.9523	0.8602	109	327	0.9891	0.9571	106	424
Ultimate Load		789				823			
Efficiency of System($R_{\beta pj}$)		Applying LMT($R_{\beta pj}$)			0.9763	Applying DPT($R_{\beta pj}$)			0.9836

7.6 Comparative Analysis of Optimization Methods for Size Using the IRRM: LMT vs. DPT

Table 14: Results Correlating the Lagrange Multiplier Method Including a Rounding-off Techniques and Dynamic Programming Approaches for Sizing in Series-Parallel Configuration Systems

		Including a Rounding Off				Dynamic Programming			
Phase	$t_{\beta j}$	$r_{\beta j}$	$R_{\beta pj}$	C_{pc}	$t_{\beta vj} \cdot C_{pc}$	$r_{\beta j}$	$R_{\beta pj}$	C_{pc}	$t_{\beta vj} \cdot C_{pc}$
I	4	0.9589	0.8625	17	68	0.9982	0.9945	20	60
II	3	0.9514	0.8745	32	96	0.9736	0.9229	38	114
III	3	0.9523	0.8602	49	147	0.9891	0.9571	58	232
Ultimate Value		311				406			
Efficiency of System($R_{\beta pj}$)		Applying LMT($R_{\beta pj}$)			0.9763	Applying DPT($R_{\beta pj}$)			0.9924

8. Analysis and Implications

In the present manuscript, the author presents a comprehensive integrated redundant reliability model that employs a series-parallel configuration to effectively determine component reliability, stage reliability, and the requisite number of components, thereby enhancing overall system reliability. The study focuses on a typical oil burner machine, which consists of a multitude of interdependent components. Among these, the author critically examines key elements such as the nozzle, fuel pump, and ignition transformer, evaluating their performance metrics in relation to the system's reliability. To achieve this, the components underwent rigorous testing across three distinct operational stages, during which their efficiencies were meticulously assessed using the Lagrange multiplier method. However, it was observed that the resultant efficiency values yielded real numbers that do not conform to practical, real-life applications; these outcomes are often deemed unacceptable in the context of reliability engineering. Ultimately, the study concludes that while the system reliability has been significantly enhanced, substantial adjustments were required for the critical components to express their efficiencies in integer terms.

This adjustment facilitated the improvement of both component and stage reliability, taking into consideration essential factors such as cost, weight, and volume. Through this integrated approach, the author not only advances the understanding of reliability modeling in complex systems but also contributes valuable insights into optimizing component performance within the constraints of real-world operational scenarios.

The computational efficiency of the proposed methods was evaluated by analyzing their complexity and CPU time. For the optimization of the Integrated Redundant Reliability (IRR) model, the Lagrange Multiplier Technique (LMT) exhibited a complexity of $O(n^2)$ with an average CPU time of 3.1 minutes, while the Dynamic Programming Technique (DPT) showed a higher complexity of $O(2^n)$ with an average CPU time of 3.4 minutes. All computations were performed using MATLAB R2022a.

9. Conclusion

The present work introduces an integrated reliability model specifically designed for a series-parallel configuration system, accommodating multiple efficiency criteria. Upon discovering that the data pertains to real numbers, the Lagrange multiplier method is employed to calculate critical parameters including the number of components $t_{\beta j}$, their respective efficiencies $r_{\beta j}$, stage reliabilities $R_{\beta pj}$, and overall system reliability $R_{\beta pj}$. The efficiencies toward worth obtained from this analysis are quantified as $r_{\beta j} = 0.9589$, 0.9514, and 0.9523, with corresponding stage reliabilities of $R_{\beta pj} = 0.8625$, 0.8745 and 0.8615. The resulting structure reliability is calculated to be $R_{\beta pj} = 0.9684$. Similarly, the efficiencies toward load obtained from this analysis are quantified as $r_{\beta j} = 0.9475$, 0.9304, and 0.9414, with corresponding stage reliabilities of $R_{\beta pj} = 0.8471$, 0.8347 and 0.8047. The resulting structure reliability is calculated to be $R_{\beta pj} = 0.9684$. Finally, the efficiencies toward dimensions obtained from this analysis are quantified as $r_{\beta j} = 0.8741$, 0.8445, and 0.8456, with corresponding stage reliabilities of $R_{\beta pj} = 0.6777$, 0.6487 and 0.5461. The resulting structure reliability is calculated to be $R_{\beta pj} = 0.9684$.

To ensure practical applicability, a dynamic programming approach is utilized to derive integer solutions, leveraging the inputs obtained from the Lagrange multiplier analysis. This results in refined for worth-component reliabilities of $r_{\beta j} = 0.9622$, 0.9601 and 0.9734, and stage reliabilities of $R_{\beta pj} = 0.8571$, 0.8849 and 0.9224. The enhanced system reliability is measured at $R_{\beta pj} = 0.9891$. Similarly, this results in refined for load-component reliabilities of $r_{\beta j} = 0.9145$, 0.9736 and 0.9891, and stage reliabilities of $R_{\beta pj} = 0.9945$, 0.9229 and 0.9571. The enhanced system reliability is measured at $R_{\beta pj}$ =0.9836. Finally, this results in refined for dimension-component reliabilities of $r_{\beta j} = 0.9982$, 0.9736 and 0.9891, and stage reliabilities of $R_{\beta pj} = 0.9945$, 0.9229 and 0.9571. The enhanced system reliability is measured at $R_{\beta pj} = 0.9924$. It is noteworthy that while adjustments to the cost, weight, and dimension of the components were minimal, these modifications led to significant improvements in stage reliability, ultimately contributing to increased system reliability at every stage as well as at each component.

The integrated reliability model (IRM) developed through this methodology proves to be exceptionally beneficial, particularly in real-world scenarios where a series-parallel configuration with reliability engineering redundancy is essential. This model is particularly advantageous for design engineers focused on dependability, especially in applications where the value of the system is constrained, enabling the selection of high-quality and efficient materials. For future research, the authors suggest employing a novel approach that restricts the minimum and maximum values of component reliability while maximizing system dependability. This can be achieved using contemporary heuristic methods to construct similar integrated reliability models with redundancy, thus enhancing their applicability in reliability engineering contexts.

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