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Determining Electrical Vehicle Charging Stations Sites Using Dominance in Neutrosophic Fuzzy Directed Graphs

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Abstract. In this paper, we begin by describing several forms of effective arcs in neutrosophic fuzzy-directed graphs, based on the fundamental principles introduced in neutrosophic fuzzy-directed graphs. We then investigate several forms of dominance in neutrosophic fuzzy directed graphs, looking at their significance and uses in decision-making situations. Our study fills two important gaps in the literature: it extends the concept of domination from fuzzy directed graphs to neutrosophic fuzzy directed graphs, and it provides a detailed characterization of dominations within this advanced framework. The concept of dominations in fuzzy graphs, fuzzy directed graphs, intuitionistic fuzzy graphs, neutrosophic fuzzy graphs, and picture fuzzy graphs is well-documented in the literature. First, we characterize several kinds of effective arcs that are particular to neutrosophic fuzzy directed graphs, such as semi-$\varkappa$ effective arc, semi-φ effective arc, and semi-ϖ effective arc. Next, we provide the notions of dominations and domination numbers connected to these arcs in different types of fuzzy-directed graphs that are neutrosophic. In particular, our analysis of minimum dominating sets provides valuable characterizations of dominations in these graphs. We also study the smallest dominating sets and dominance numbers for neutrosophic fuzzy dipaths and dicycles, and we provide some interesting results. Finally, we suggest an algorithm to handle decision-making problems, such figuring out the ideal places to launch EV charging station in different metropolitan regions, by utilizing the ideas presented in this research. In regard to the development of electric mobility and environmentally friendly urban growth, this strategy offers a far superior technique of identifying EV charging stations.

2020 Mathematics Subject Classifications: 05C72, 05C69, 05C90, 68R10, 90B90

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1. Introduction

Zadeh initiated the theory of FSs in 1965 [54] as the development of the classical theory of sets. FSs have been used subsequently in different areas, such as computer

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science, management, artificial intelligence, and decision making because they have superior capability in handling with the large amount of information in contrary to the classic logic. Due to the flexibility for source selection, there have been a number of extensions to FSs. Interval-valued fuzzy sets (IVFSs) are among the first generalizations that Zadeh originally introduced in 1975 [55]. As opposed to FSs where membership values are single numbers in the range [0,1], IVFSs permit the membership grade to be a subinterval of [0,1]. However, non-membership was not explicitly explored in either FSs or IVFSs. Atanassov [14] proposes the concept of non-membership and starts the study of the IFSs, where the membership and non-membership are two independent factors integrated considering the fact that the sum of membership and non-membership should not be over 1. In the past few decades, Graph theory has emerged as one powerful tool for analyzing real-world problems across divergent sciences. Various kinds of graphical models have been used in various branches of studies including pattern management, computer networking, and decision making. Fuzzy logic grows from uncertainty in real-world problems and has developed into a useful and potent resolution. In recent years, a new approach to model uncertainty is known as fuzzy graphs (FGs). First proposed by Rosenfeld [42], FGs are considered more efficient and elastic in comparison with classical graphs. Because of this flexibility, Their applications have been investigated in numerous fields. Some new terms to the FG theory were introduced by Bhattacharya [15], while advanced operation was defined in [32]. The complement of FGs was for the first time in [50], whereas, the average connectivity in the FGs has been described by Poulik et al. [37]. Most of the authors have generalized the classical graph concepts towards FGs that has led to the use of this concept in areas like social studies and road network analysis. Later improvements consist of the extension of aggregation operators in a complicated fuzzy setting by Ubaid ur Rehman [51]; Jana et al. [23] proposed the Dombi operator using Pythagorean fuzzy knowledge under multi-attribute decision-making. The edge coloring of FGs has been investigated by Mahapatra et al. [28]. A new concept called fuzzy-directed graphs (FDGs) was defined by Deepak Mordeson and developed in [33] and [26]. Bipolar FDGs and their applications in decision-making are explored in [3]. The generalized forms of FGs have been defined as intuitionistic fuzzy graphs (IFGs) in [46]. Similarly, more elaborate IFGs with networking applications were presented in [53]. The dynamics of IFGs are explained by [45] through strong and effective arcs only; categorical properties of IFGs are introduced by [41]. Competition graphs have been defined by Akram et al. [7] under a complex intuitionistic fuzzy environment and later extended in terms of IF-hypergraphs, Strong-IFGs, IF-cycles, and IF-trees [[6]-[4]]. Complex intuitionistic fuzzy power interaction aggregation operators by Ali et al. [13], on decision-making techniques. The use of IFDGs for a decision support system was considered in [5], and interval valued IFDG matrix for roadblock analysis in [47]. Bipolar IFDGs were initialised using energy by Nithyanandham et al. [36] and applied to decision-making.

Recent years witnessed extensive interest in neutrosophic graphs since they can deal with imprecise, uncertain, and indeterminate data, which are so characteristic of the current complex-world problems. Neutrosophic logic, which generalized the classical and fuzzy logic by involving truth, uncertainty, and false parts, has given sufficient mathematical support to various graph-oriented studies. An extended work of domination in neutrosophic graphs is being studied, which involves various applications in areas including decision-making, communication network, and computational intelligence. Mullai and Broumi examined the notion of dominating energy in neutrosophic graphs, which can help to design efficient energy networks [35]. Fei used neutrosophic graphs to wireless networks, proving the efficiency of application of NM to handle the uncertainties of communication networks [20]. The recent study conducted by Khan et al. on covering and paired domination under the neutrosophic context has extended the broad spectrum application of neutrosophic graphs in [25], Similarly, in single-valued neutrosophic graphs, the authors analyzed complementary domination and edge domination in the related field of graph-based analysis [[52],[29]]. A few recent works have been mainly targeted at examining different operations of and using various kinds of neutrosophic graphs, such as Dombi neutrosophic graphs, which have been discussed by Lakhwani et al., [27]and domination in neutrosophic incidence graphs as concepts further developed by Mohamad et al., [31]. Secure dominance, integrity, and domination integrity in neutrosophic graphs were also attempted, by Broumi and Jaikumar et al. (2023) [[16],[22]]. Other extensions to the field of the present work include further studies in fermatean neutrosophic Dombi fuzzy graphs, which have been studied by Sasikala and Divya [43], and offered new insights into the aspect of neutrosophic graph theory. Furthermore, by extension complex fermatean neutrosophic graphs have been used as showcased by Broumi et al. [17] for decision-making purposes. Subsequent studies of equitable domination in neutrosophic graphs focusing on strong arcs by Meenakshi and Senbagamalar [30], as well as new definitions of domination in neutrosophic incidence graphs by Smarandache et al. [48], have extended the theoretical framework of the subject matter. The existence of domination of neutrosophic over graphs as studied by [18], NarmadaDevi and the use of neutrosophic networks applying efficient domination as described by Senbagamalar et al. [44] establishes the practical importance for neutrosophic graphs in handling decision-making dilemmas in conditions of uncertainty. In continuation with their work, Kaviyarasu et al. [24]have attempted to forward circular economy strategies for sustainable development concerning t-neutrosophic fuzzy graphs. This study points to the applicability of neutrosophic fuzzy graph models improves the problematic determination in the circular economy by managing all the large resources and wastes effectively. The use of neutrosophic fuzzy logic here shows that these models could be used to solve complex sustainability issues.

Furthermore, neutrosophic graphs are useful in infrastructure design. Alqahtani et al. [21] used complex neutrosophic graphs to design the structure of the hospital by incorporating multi-constrained and multi-uncertain factors into the planning model. In the same vein, AL-Omeri and Kaviyarasu [9] discussed how the concept of neutrosophic graphs can be applied for the operational enhancement of earthquake response centers, and how such mathematical models can better capture uncertain and variable environmental conditions that affect disaster response planning and execution. Such applications reveal the increasing importance of neutrosophic fuzzy graph theory in modeling the current problems. For neutrosophic graphs, [12] also developed survey results of planer and outer planar, suggesting a direction of further topological structure investigation of this framework. That is, Sreelakshmi and Samundesvari [49]reported the concept of total domination of USVNG in 2024 to expand the consideration of dominance theory in the context of neutrosophic systems. In [39] This article explores multi-objective fuzzy programming, which may relate to the decision-making framework used in neutrosophic fuzzy graph theory. In [1] focus on parameter estimation techniques can inform the modeling and decision-making processes in the EV station framework. [2]The mathematical modeling approach used here could provide conceptual or technical inspiration for modeling domination in neutrosophic fuzzy directed graphs. [38] The decision-making framework addressing under constraints and uncertainties could be linked to EV charging locations under various constraints. [40] Directly relevant due to its focus on neutrosophic methods, which align well with the methodology in your paper. The set of works by Al-Masarwah and other authors gives important contributions to fuzzy mathematics and its use in different fields [10]. The earlier studies provide an initial understanding of fuzzy soft graph essentials, including some elements and operations concerning fuzzy uncertain graph models [11]. The subsequent works are devoted to further development of the sophisticated classification of different types of fuzzy soft graphs and the explorations of further mathematical properties of the fuzzy soft graphs, which enhances the theory. The other extensions extend the above studies by defining m-polar fuzzy points in BCK/BCI-algebras, which links fuzzy logic and algebraic systems [8]. This paper also introduces int-soft ideals in ordered semigroups, it expand these ideas to ideal theory to provide further development for algebraic modeling [34].

In this study, we also define several new concepts of dominations in neutrosophic fuzzy directed graphs(NFDGs) by using the concepts of the different types of effective arcs. An algorithm created to address real-world issues, like figuring out the best places for EV charging stations, demonstrates the work's practical significance. This method improves the efficacy and efficiency of decision-making procedures in transportation networks and urban planning by utilizing the special qualities of NFDGs. This study illustrates the adaptability and usefulness of NFDGs in tackling contemporary issues by bridging the gap between theoretical developments and real-world implementations. list of Abbreviations given in Table 1.

1.1. Motivation

Different from the fuzzy directed graphs (FDGs) where only the degree of membership associated with the directed edges is taken into consideration, in neutrosophic fuzzy directed graphs (NFDGs) the degree of truth, indeterminacy and falsity attached to the edges are taken into account, making NFDGs more general and flexible than FDGs. Further, the domination concept has been introduced in fuzzy graphs (FGs), interval-valued fuzzy graphs (IFGs), and Pythagorean fuzzy graphs (PFGs), and hence it inspired us to define the domination concepts in NFDGs and explain their usefulness in decision-making.

Table 1: List of Abbreviations

Abbreviations	**Meaning**
FDGs	Fuzzy Directed Graphs
NFDGs	Neutrosophic Fuzzy Directed Graphs
PFGs	Pythagorean Fuzzy Graphs
FGs	Fuzzy Graphs
IFGs	Intuitionistic Fuzzy Graphs
NF-dipath	Neutrosophic Fuzzy Dipath
NF-dicycle	Neutrosophic Fuzzy Dicycle
MD	Membership degrees
NMD	Non Membership degrees
EV	Electrical Vehicle
$\mathcal{DS}$	Dominating Set
$\mathcal{MDS}$	Minimal Dominating Set

1.2. Novelty

(i) In the case of NFDGs, we also discussed other types of effective arcs, including semi-$\varkappa$ effective arcs, semi-φ effective arcs, and semi-ϖ effective arcs provided adequate definitions for these terms.

(ii) Finally, a Dominating set, Minimal dominating set ($\mathcal{MDS}$) and domination number were studied in relation to certain defined NFDGs.

(iii) Similarly, we defined neutrosophic fuzzy dipath (NF-dipath) and neutrosophic fuzzy dicycle (NF-dicycle) in NFDGs and stated their domination-dependent characteristics.

(iv) Last but not least, we described both theoretical and practical aspects of the above-mentioned concepts in decision-making situations.

1.3. Structure of the article

This manuscript has six sections. Section 2 discusses key terminologies and concepts associated with neutrosophic sets, fuzzy graphs, and their generalizations. Section 3 explains different kinds of useful arcs in NFDG and the domination concepts that depend on them. In section 4 the author tackles a decision-making problem and shows how domination works in NFDGs. Section 5 consists of a comparative analysis, and Section 6 contains the last comments including suggestions for future research.

2. Preliminaries

Definition 1. *[54] If Z is a nonempty set and $\lambda : Z \to [0, 1]$ is the membership function, then (λ, Z) is a fuzzy set.*

Definition 2. *[31] An generalisation of a fuzzy set, a neutrosophic fuzzy set consists of three independent membership functions for each element* ρ *in a universe* U*: falsity-membership* $\varpi(\rho)$*, truth-membership* $\varkappa(\rho)$*, and indeterminacy-membership* $\varphi(\rho)$*. These functions meet the requirement:* $\varkappa(\rho), \varphi(\rho), \varpi(\rho) \in [0,1]$ *and* $\varkappa(\rho) + \varphi(\rho) + \varpi(\rho) \leq 3$

Definition 3. *The dominant set (*$\mathcal{DS}$*) in* $\tilde{G}$ *is a subset* X_1 *of* X*, provided that for any* $w \in X_1$*, there exists* $\rho \in U - X_1$ *such that* w *dominates* ρ*. If* X_1 *contains no suitable* $\mathcal{DS}$ *of* $\tilde{G}$*, then a* $\mathcal{DS}$ X_1 *of a fuzzy graph* $\tilde{G}$ *is a minimum dominating set (*$\mathcal{MDS}$*). The lowest (fuzzy) cardinality of a* $\mathcal{DS}$ *in* $\tilde{G}$ *is its dominant number (*$\mathcal{DN}$*).*

Definition 4. *[19] Consider the basic directed graph* $\widehat{G}_D = (V, E)$*, where* V *is a nonempty finite set and* $E = \{(\rho_1, \rho_2) \mid \rho_1, \rho_2 \in V, \rho_1 \neq \rho_2\}$*. The pair* $\tilde{G}_D = (\mu_V, \mu_E)$ *is a fuzzy digraph (FDG), where* $\mu_V : V \to [0,1]$ *and* $\mu_E : E \to [0,1]$ *with* $\mu_E((\rho_1, \rho_2)) \leq \mu_V(\rho_1) \wedge \mu_V(\rho_2)$*, for all* $\rho_1, \rho_2 \in V$*.*

Definition 5. *[19] Consider a hidden digraph* $\widehat{G}_D = (V, E)$*, in which* V *is the set of vertices and* E *is the set of directed edges. Hence,* $\tilde{G}_D = (\mu_V, \mu_E)$ *is the fuzzy directed graph of* $\widehat{G}_D$*, where* $\mu_V = \{\mu_V(\rho_1) \mid \rho_1 \in V\}$ *is the set of vertices (nodes), and* $\mu_E = \{\mu_E((\rho_1, \rho_2)) \mid \rho_1, \rho_2 \in V\}$ *represents the fuzzy directed edges from* $\mu_V(\rho_1)$ *to* $\mu_V(\rho_2)$ *in a fuzzy digraph* $\tilde{G}_D$*.*

Definition 6. *[19] A hidden digraph of an FDG* $\tilde{G}_D = (\mu_V, \mu_E)$ *is represented as* $\widehat{G}_D = (V, E)$*. The arc* $(\rho_1, \rho_2) \in E$ *is thus referred to be an effective arc of* $\tilde{G}_D$*, if* $\mu_E(\rho_1, \rho_2) \leq \mu_V(\rho_1) \wedge \mu_V(\rho_2)$*, for all* $\rho_1, \rho_2 \in V$*.*

3. Dominance via Effective Arcs in the NFDGs

First, we introduce several kinds of effective edges in NFDGs, such as semi-$\varkappa$ effective arcs, semi-φ effective arcs, and semi-ϖ effective arcs etc . Using these arcs, we further develop these ideas to present the idea of domination in NFDGs. Within this framework, we analyze several significant domination-related NFDG features. We also provide some interesting findings on Minimum $\mathcal{MDS}$ and $\mathcal{DS}$ in IFDGs.

In addition, we define the terms "NF-dipath" and "NF-dicycle" inside NFDGs and investigate the $\mathcal{MDS}$s, Dominating Numbers (DNs), and related ideas that are connected to them. We also provide some interesting findings about NF-dicycles and NF-dipaths in NFDGs.

Throughout the discussion, $\mathcal{G}_\Gamma = (\Lambda, \Omega)$ will signify an NFDG of a hidden digraph $\hat{\mathcal{G}}_\Gamma = (V, E)$, where $\Lambda = (\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda)$ and $\Omega = (\varkappa_\Omega, \varphi_\Omega, \varpi_\Omega)$. To begin, we define an effective arc in an NFDG as follows:

Definition 7. *An arc (x, y) in an NFDG* $\mathcal{G}_\Gamma$ *is considered an effective arc if it meets the requirements listed below:* $\varkappa_\Omega(x, y) = \min\{\varkappa_\Lambda(x), \varkappa_\Lambda(y)\}$ $\varphi_\Omega(x, y) = \min\{\varphi_\Lambda(x), \varphi_\Lambda(y)\}$ *and* $\varpi_\Omega(x, y) = \max\{\varpi_\Lambda(x), \varpi_\Lambda(y)\}$ *where* $\varkappa_\Omega(x, y) + \varphi_\Omega(x, y) + \varpi_\Omega(x, y) \leq 3$ *for all* $(x, y) \in \Omega$*. Next, we delineate the related categories of effective arcs in an NFDG, namely the semi-*$\varkappa$ *effective arcs, semi-*φ *effective arcs, and semi-*ϖ *effective arcs*

Definition 8. *A semi-$\varkappa$ effective arc of the NFDG $\mathcal{G}_\Gamma$ is defined as,*

$$\begin{aligned} \varkappa_\Omega(x,y) &= \min\{\varkappa_\Lambda(x),\varkappa_\Lambda(y)\} \\ \varphi_\Omega(x,y) &\neq \min\{\varphi_\Lambda(x),\varphi_\Lambda(y)\} \\ \text{and } \varpi_\Omega(x,y) &\neq \max\{\varpi_\Lambda(x),\varpi_\Lambda(y)\} \end{aligned}$$

with $\varkappa_\Omega(x,y)+\varphi_\Omega(x,y)+\varpi_\Omega(x,y)\leq 3$ for all $(x,y)\in\Omega$.

Definition 9. *A semi-φ effective arc of the NFDG $\mathcal{G}_\Gamma$ is defined as,*

$$\begin{aligned} \varkappa_\Omega(x,y) &\neq \min\{\varkappa_\Lambda(x),\varkappa_\Lambda(y)\} \\ \varphi_\Omega(x,y) &= \min\{\varphi_\Lambda(x),\varphi_\Lambda(y)\} \\ \text{and } \varpi_\Omega(x,y) &\neq \max\{\varpi_\Lambda(x),\varpi_\Lambda(y)\} \end{aligned}$$

with $\varkappa_\Omega(x,y)+\varphi_\Omega(x,y)+\varpi_\Omega(x,y)\leq 3$ for all $(x,y)\in\Omega$.

Definition 10. *A semi-ϖ effective arc of the NFDG $\mathcal{G}_\Gamma$ is defined as,*

$$\begin{aligned} \varkappa_\Omega(x,y) &\neq \min\{\varkappa_\Lambda(x),\varkappa_\Lambda(y)\} \\ \varphi_\Omega(x,y) &\neq \min\{\varphi_\Lambda(x),\varphi_\Lambda(y)\} \\ \text{and } \varpi_\Omega(x,y) &= \max\{\varpi_\Lambda(x),\varpi_\Lambda(y)\} \end{aligned}$$

with $\varkappa_\Omega(x,y)+\varphi_\Omega(x,y)+\varpi_\Omega(x,y)\leq 3$ for all $(x,y)\in\Omega$.

Example 1. *Concerning Fig. 1 NFDG, we have the following.*

- *(i) Examine the arc (b,c), In this case $\varkappa_\Omega(b,c)=\min\{\varkappa_\Lambda(b),\varkappa_\Lambda(c)\}=\min(0.4,0.7)=0.4$, $\varphi_\Omega(b,c)=\min\{\varphi_\Lambda(b),\varphi_\Lambda(c)\}=\min(0.5,0.9)=0.5\neq 0.6$, and $\varpi_\Omega(b,c)=\max\{\varpi_\Lambda(b),\varpi_\Lambda(c)\}=\max(0.5,0.7)=0.7\neq 0.4$. As a result, an arc (b, c) is non-effective, whereas an arc (b, c) is semi-$\varkappa$ effective.*

- *(ii) Examine the arc (c,d), In this case $\varkappa_\Omega(c,d)=\min\{\varkappa_\Lambda(c),\varkappa_\Lambda(d)\}=\min(0.7,0.8)=0.7\neq 0.4$, $\varphi_\Omega(c,d)=\min\{\varphi_\Lambda(c),\varphi_\Lambda(d)\}=\min(0.5,0.6)=0.5$, and $\varpi_\Omega(c,d)=\max\{\varpi_\Lambda(c),\varpi_\Lambda(d)\}=\max(0.5,0.5)=0.5\neq 0.3$. As a result, an arc (c, d) is non-effective, whereas an arc (c, d) is semi-φ effective.*

- *(iii) Examine the arc (a,b), In this case $\varkappa_\Omega(a,b)=\min\{\varkappa_\Lambda(a),\varkappa_\Lambda(b)\}=\min(0.5,0.4)=0.4\neq 0.3$, $\varphi_\Omega(a,b)=\min\{\varphi_\Lambda(a),\varphi_\Lambda(b)\}=\min(0.6,0.9)=0.6\neq 0.5$, and $\varpi_\Omega(a,b)=\max\{\varpi_\Lambda(a),\varpi_\Lambda(b)\}=\max(0.8,0.7)=0.8$. As a result, an arc (a, b) is non-effective, whereas an arc (a, b) is semi-ϖ effective.*

- *(iv) Examine the arc (d,a), In this case $\varkappa_\Omega(d,a)=\min\{\varkappa_\Lambda(d),\varkappa_\Lambda(a)\}=\min(0.5,0.8)=0.5$, $\varphi_\Omega(d,a)=\min\{\varphi_\Lambda(d),\varphi_\Lambda(a)\}=\min(0.6,0.6)=0.6$, and $\varpi_\Omega(d,a)=\max\{\varpi_\Lambda(d),\varpi_\Lambda(a)\}=\max(0.8,0.5)=0.8$. As a result, an arc (d, a) is an effective arc.*

Definition 11. *Let $\mathcal{G}_\Gamma=(\Lambda,\Omega)$ will signify an NFDG of a hidden digraph $\hat{\mathcal{G}}_\Gamma=(V,E)$, then*

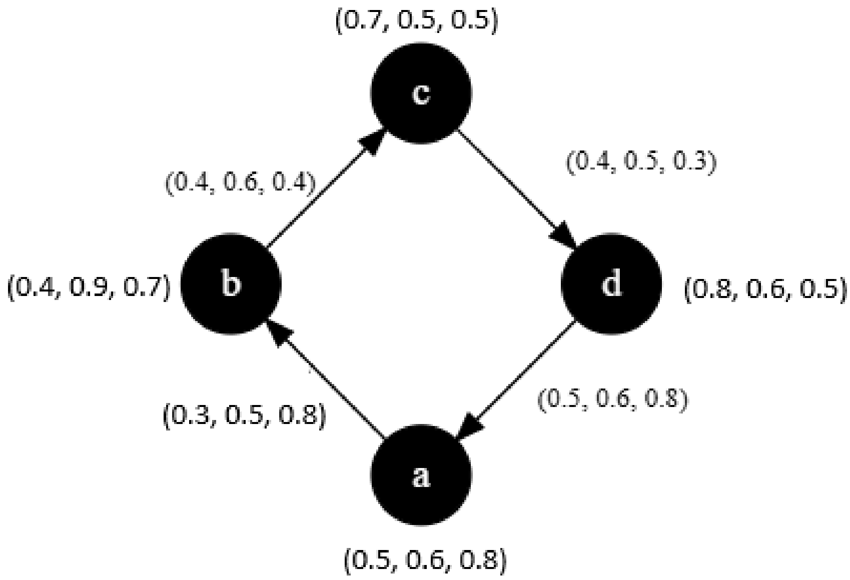

Figure 1: NFDG .

(i) The effective neighborhood (EN) of a vertex $x \in V$ is defined as

$$\daleth_E(x) = \{y \in V \mid arc(x, y) \text{ is an effective arc}\},$$

while the closed effective neighborhood (CEN) of y is given by

$$\daleth_E[x] = \daleth_E(x) \cup \{x\}.$$

(ii) The semi-$\varkappa$ EN of $x \in V$ is defined as

$$\daleth_{\varkappa E}(x) = \{x \in V \mid arc(x, y) \text{ is a semi-}\varkappa\text{-effective arc}\},$$

and the closed semi-$\varkappa$-EN of x is

$$\daleth_{\varkappa E}[x] = \daleth_{\varkappa E}(x) \cup \{x\}.$$

(iii) The semi-φ EN of $x \in V$ is defined as

$$\daleth_{\varphi E}(x) = \{x \in V \mid arc(x, y) \text{ is a semi-}\varphi\text{-effective arc}\},$$

and the closed semi-φ-EN of x is

$$\daleth_{\varphi E}[x] = \daleth_{\varphi E}(x) \cup \{x\}.$$

(iv) *The semi-ϖ EN of $x \in V$ is defined as*

$$\daleth_{\varpi E}(x) = \{x \in V \mid arc(x, y) \text{ is a semi-}\varpi\text{-effective arc}\},$$

and the closed semi-ϖ-EN of x is

$$\daleth_{\varpi E}[x] = \daleth_{\varpi E}(x) \cup \{x\}.$$

(v) *The minimum cardinality of effective neighborhoods in a graph $\mathcal{G}_\Gamma$ is*

$$\delta_E(\mathcal{G}_\Gamma) = \min\{|\daleth_E(x)| \mid x \in V(\mathcal{G}_\Gamma)\}.$$

(vi) *The maximum cardinality of effective neighborhoods in a graph $\mathcal{G}_\Gamma$ is*

$$\Delta_E(\mathcal{G}_\Gamma) = \max\{|\daleth_E(x)| \mid x \in V(\mathcal{G}_\Gamma)\}.$$

Definition 12. *Let $\mathcal{G}_\Gamma$ be an NFDG with two vertices, x and y. The vertex x is said to dominate the vertex y if the directed edge (x, y), is considered an effective edge.*

Example 2. *The directed edge (d, a) in the NFDG depicted in Fig. 1 is a effective edge, however, the edges $(a, b), (b, c)$ and (c, d) are not effective arcs. Because of the effective edge, the vertex d dominates the vertex a. However, vertex a does not dominate b, vertex b does not dominate c, and vertex c does not dominate d as there are no effective edges between them.*

Definition 13. *In an NFDG, U is a subset of V and is considered a $\mathcal{DS}$ if for each $y \in V - U$ there exists $x \in V$ such that x dominates y.*

Definition 14. *A dominating set U is called a $\mathcal{MDS}$ in an NFDG if it contains no other dominating sets. The effective arc domination number (DN), represented by $\zeta_E(\mathcal{G}_\Gamma)$, is the minimal fuzzy cardinality among all dominating sets in an NFDG. The minimal effective edge dominating set's total membership count is provided by $n(\zeta_E(\mathcal{G}_\Gamma))$.*

Example 3. *From Fig. 2, it is easy to infer that the $\mathcal{MDS}$s of an NFDG $\mathcal{G}_\Gamma$ are represented by $\{(0.9, 0.6, 0.2), (0.8, 0.5, 0.3)\}$ and $\{(0.8, 0.5, 0.3), (0.6, 0.3, 0.5)\}$.*

Definition 15. *Let $\mathcal{G}_\Gamma = (\Lambda, \Omega)$ will signify an NFDG. The only $\mathcal{DS}$ of $\mathcal{G}_\Gamma$ is $(\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda)$ if*

$$\begin{aligned} \varkappa_\Omega(x, y) &\leq \min\{\varkappa_\Lambda(x), \varkappa_\Lambda(y)\} \\ \varphi_\Omega(x, y) &\leq \min\{\varphi_\Lambda(x), \varphi_\Lambda(y)\} \\ \text{and } \varpi_\Omega(x, y) &\leq \max\{\varpi_\Lambda(x), \varpi_\Lambda(y)\} \end{aligned}$$

Example 4. *It is straightforward to confirm that $\{(0.5, 0.6, 0.8), (0.4, 0.9, 0.7), (0.9, 0.6, 0.2), (0.7, 0.5, 0.5), (0.8, 0.6, 0.5)\}$ is the sole $\mathcal{DS}$ of an NFDG $\mathcal{G}_\Gamma$ shown in Fig. 3.*

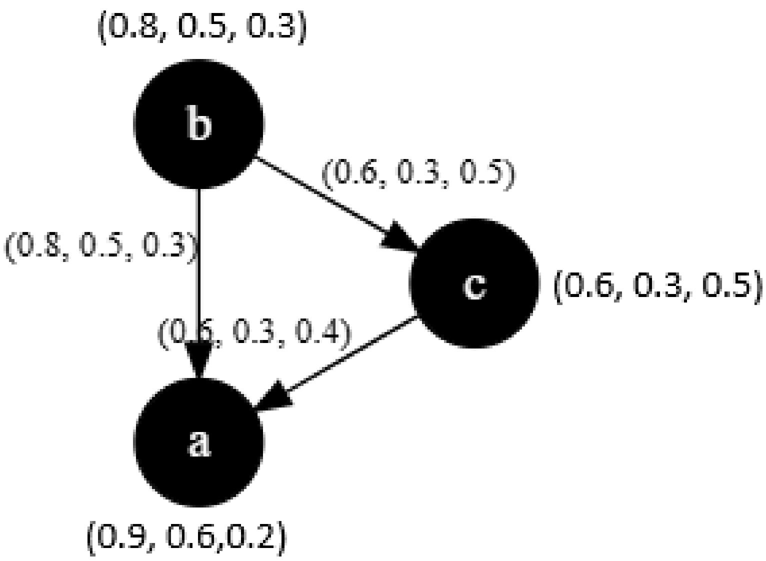

Figure 2: $\mathcal{MDS}$.

Definition 16. *Let $\mathcal{G}_\Gamma$ be an NFDG and let x, y be its two vertices.*

(i) x (semi-$\varkappa$-effective) dominates y, if the arc (x, y) is semi-$\varkappa$-effective.

(ii) x (semi-φ-effective) dominates y, if the arc (x, y) is semi-φ-effective.

(iii) x (semi-ϖ-effective) dominates y, if the arc (x, y) is semi-ϖ-effective.

Example 5. *It is clear from Fig.1 that the arc (a, b) is a semi-(ϖ) effective arc, indicating that a semi-(ϖ)-effectively dominates b. In the same way, the arc (b, c) is a semi-$(\varkappa)$-effective arc; as a result, b semi-$(\varkappa)$-effectively dominates c, and the arc (c, d) is a semi-(φ)-effective arc; as a result, c semi-(φ)-effectively dominates d.*

Theorem 1. *Let $\mathcal{G}_\Gamma = (\Lambda, \Omega)$ be an NFDG, and let $\hat{\mathcal{G}}_\Gamma = (V, E)$ be its hidden digraph with $\aleph \subseteq V$. A $\mathcal{DS}$ $\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)$ of $\mathcal{G}_\Gamma$ is minimal if and only if for each $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e) \in \varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$, either*

$$(\daleth_{\mathcal{G}_\Gamma})(\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) \cap (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) = \emptyset$$

or

$$(\daleth_{\mathcal{G}_\Gamma})(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \cap (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) = \{\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)\},$$

for some $(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \in (\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda) \setminus (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$.

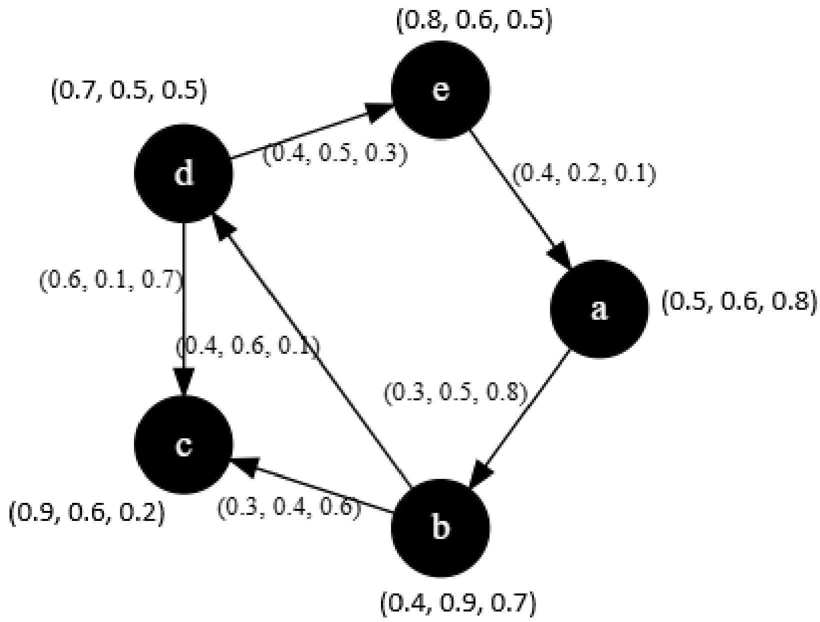

Figure 3: Dominating set in NFDG .

Proof. Let $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) \in (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$ such that $(\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$ is an $\mathcal{MDS}$ of $\mathcal{G}_\Gamma$, then

$$(\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) \setminus (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$$

. Hence

$$(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \notin (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) \setminus (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$$

Consider that $(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f))$ is not dominated by any of the members of $(\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) \setminus (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$.
Now, we have the following cases:
Case 1. Let $(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) = (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$.
Then $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$ is not dominated by any of the members of $(\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) \setminus (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$, i.e.,
$(\urcorner_{\mathcal{G}_\Gamma})((\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))) \cap (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) = \emptyset$.
$Case 2. Let (\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \neq (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$. Then $(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \notin (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$. Since $(\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$ is an $\mathcal{MDS}$ of $\mathcal{G}_\Gamma$, it implies that $(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f))$ is dominated by $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) \in (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$.
Thus,

$$(\urcorner_{\mathcal{G}_\Gamma})(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \cap (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph)) = \{(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))\},$$

$for some (\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \in (\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda) \setminus (\varkappa_\Lambda(\aleph), \varphi_\Lambda(\aleph), \varpi_\Lambda(\aleph))$. Likewise, its opposite may also be demonstrated.

Definition 17. *NF-dipaths $\mathcal{D}_p$ are effective arcs where the starting node of the next edge is the same as its ending node.*

Consider $D_p = (\Lambda, \Omega)$ as an NF-dipath of a hidden dipath $D_n = (V, U)$, where n denotes an integer with $l \geq 2$. Then:
1.$(\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda) = \{(\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k)) : e_k \in \Lambda,$ for all $k \in \{1, 2, \dots, l\}\}$.
2.$(\varkappa_\Omega, \varphi_\Omega, \varpi_\Omega) = \{(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1}) : (e_k, e_{k+1}) \in \Lambda,$ for all $k \in \{1, 2, \dots, l-1\}\}$.
3.$(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1}) = \{(\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k)), (\varkappa_\Lambda(e_{k+1}), \varphi_\Lambda(e_{k+1}), \varpi_\Lambda(e_{k+1}))\}$, for all $k \in \{1, 2, \dots, l-1\}$.
4. The start and end vertices of a nontrivial NF-dipath are $(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1))$ *and* $(\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l))$, *respectively.*

Theorem 2. *An NF-dipath is represented by D_p. Following that, one of the prerequisites needs to be met:*
1. $\zeta(D_p) = \sum_{r=1}^{l/2} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1}))$
2. $\zeta(D_p) = \min E$, *where*
$$E = \left\{ \frac{l+1}{2} - \sum_{r=1}^{(l+1)/2-r} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \dots \right.$$
$$\left. \sum_{r=(l+3)/2-k}^{(l+1)/2} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})) : k \in \{0, 1, 2, \dots, (l-1)/2\} \right\}.$$

Proof. From the Remark 3
$(\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda) = \{(\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k)) : e_k \in \Lambda$, for all $k \in \{1, 2, \dots, l\}\}$.
$(\varkappa_\Omega, \varphi_\Omega, \varpi_\Omega) = \{(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1}) : (e_k, e_{k+1}) \in \Lambda$, for all $k \in \{1, 2, \dots, l-1\}\}$. Because $(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1})$ is an effective edge for all $k \in \{1, 2, \dots, (l-1)\}$, *it implies* $(\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k))$ dominate $(\varkappa_\Lambda(e_{k+1}), \varphi_\Lambda(e_{k+1}), \varpi_\Lambda(e_{k+1}))$ for all $k \in \{1, 2, \dots, (l-1)\}$.

Case 1: If $l = 2r$ i.e., even, for positive integer r, then $\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \dots, (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}$ is clearly the $\mathcal{MDS}$ of D_p. Note that
$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \dots, (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}$
$= \sum_{r=1}^{l/2} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1}))$.
Therefore, $\zeta(D_p) = \sum_{r=1}^{l/2} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1}))$. This proves (1).

Case 2: When l = 2r- 1, when l is odd, for any integer r. Thus,

$$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \dots, (\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l))\};$$

$$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \dots, (\varkappa_\Lambda(e_{l-2}), \varphi_\Lambda(e_{l-2}), \varpi_\Lambda(e_{l-2})), (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\};$$

$$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \dots, (\varkappa_\Lambda(e_{l-4}), \varphi_\Lambda(e_{l-4}), \varpi_\Lambda(e_{l-4})), (\varkappa_\Lambda(e_{l-3}), \varphi_\Lambda(e_{l-3}), \varpi_\Lambda(e_{l-3})), (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\};$$

$\vdots$

$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \dots$

$, (\varkappa_\Lambda(e_{l-5}), \varphi_\Lambda(e_{l-5}), \varpi_\Lambda(e_{l-5})), (\varkappa_\Lambda(e_{l-3}), \varphi_\Lambda(e_{l-3}), \varpi_\Lambda(e_{l-3})), (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}.$

$\mathcal{MDS}$s for D_p. Take note of that,

$$\begin{aligned}&(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \ldots, (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\\&= \sum_{r=1}^{\frac{l+1}{2}} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})).\end{aligned}$$

Generally, we have

$$\begin{aligned}&(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)) + (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)) + \cdots + (\varkappa_\Lambda(e_g), \varphi_\Lambda(e_g), \varpi_\Lambda(e_g))\\&+ (\varkappa_\Lambda(e_{g+1}), \varphi_\Lambda(e_{g+1}), \varpi_\Lambda(e_{g+1})) + \cdots + (\varkappa_\Lambda(e_{l-3}), \varphi_\Lambda(e_{l-3}), \varpi_\Lambda(e_{l-3})) + (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\\&= \sum_{r=1}^{\frac{g+1}{2}} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \sum_{r=\frac{g+3}{2}}^{\frac{l+1}{2}} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})).\end{aligned}$$

$$\text{Let} \quad \frac{k+1}{2} = \frac{m+1}{2} \quad \text{for} \quad r \in \left\{0, 1, 2, \ldots, \frac{l-1}{2}\right\}.$$

$$\begin{aligned}&(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)) + (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)) + \cdots + (\varkappa_\Lambda(e_g), \varphi_\Lambda(e_g), \varpi_\Lambda(e_g))\\&+ (\varkappa_\Lambda(e_{g+1}), \varphi_\Lambda(e_{g+1}), \varpi_\Lambda(e_{g+1})) + \cdots + (\varkappa_\Lambda(e_{l-3}), \varphi_\Lambda(e_{l-3}), \varpi_\Lambda(e_{l-3})) + (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\\&= \sum_{r=1}^{\frac{l+1}{2}-k} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \sum_{r=\frac{l+3}{2}-k}^{\frac{l+1}{2}} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})).\end{aligned}$$

Consequently, the minimum of
$E = \left\{ \frac{l+1}{2} - \sum_{r=1}^{(l+1)/2-r} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \sum_{r=(l+3)/2-k}^{(l+1)/2} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})) \right.$
$\left. k \in \{0, 1, 2, \ldots, (l-1)/2\} \right\}$ represents a DN of an NF-dipath D_p.
It follows that $\zeta(D_p) = \min E$.

Example 6. *A hidden directed route $P_6 = (V, C)$ has an NF-dipath $D_p = (\Lambda, \Omega)$. Next, the $\mathcal{MDS}$ of D_p may be obtained from Fig.4.*

$$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\}$$

and the DN is, $\zeta(D_p) = \sum_{r=1}^{3} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1}))$.

Example 7. *From Fig. 5, let $D_p = (\Lambda, \Omega)$ be a NF-dipath of a hidden directed path $P_5 = (V, C)$. Let:*

$$A = \{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\},$$

$$B = \{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\},$$

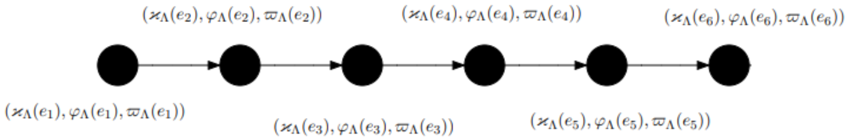

Figure 4: NFDP .

$$C = \{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\}.$$

Then, the $\mathcal{MDS}$ of D_p is $\{A, B, C\}$, and the DN is:

$$\zeta(D_p) = \min\left\{\sum_{(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) \in A} (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)), \sum_{(\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)) \in B} (\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f)), \sum_{(\varkappa_\Lambda(h), \varphi_\Lambda(h), \varpi_\Lambda(h)) \in R} (\varkappa_\Lambda(h), \varphi_\Lambda(h), \varpi_\Lambda(h))\right\}.$$

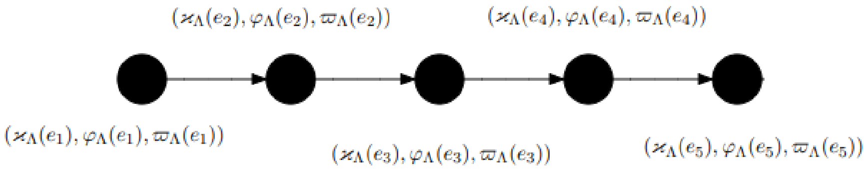

Figure 5: NFDP .

Theorem 3. *Let $D_p = (\Lambda, \Omega)$ be a NF-dipath of a hidden directed path $P_N = (V, C)$. If $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) = (\varkappa_\Lambda(f), \varphi_{Lambda}(f), \varpi_{Lambda}(f))$, then*

$$\zeta(D_p) = \frac{l}{2}(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)), \quad \forall\, e, f \in V.$$

Proof. 1. If n is even: By Theorem 2, we have:

$$\zeta(D_p) = \sum_{r=1}^{n/2} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})).$$

Since $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) = (\varkappa_\Lambda(f), \varphi_{Lambda}(f), \varpi_{Lambda}(f))$ for all $e, f \in V$, it implies that:

$(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)) = (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)) = \cdots = (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1})) =$

$(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$.

Thus,

$$\zeta(D_p) = \sum_{k=1}^{l/2} (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) = \frac{n}{2}(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

2. If m is odd: By Theorem 2, we have:

$$\zeta(D_p) = \sum_{r=1}^{(m+1)/2-k} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \sum_{r=(l+3)/2-k}^{(l+1)/2} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})),$$

for all $k \in \{0, 1, 2, \ldots, l\}$. Simplifying, we have:

$$\zeta(D_p) = \left(\frac{l+1}{2} - j\right)(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) + \left[\frac{l+1}{2} - \left(\frac{l+3}{2} - k + 1\right)\right](\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

This further simplifies to:

$$\zeta(D_p) = \frac{l+1}{2}(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

Hence, $\zeta(D_p)$ is either $\frac{l}{2}(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$ if l is even, or $\frac{l+1}{2}(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$ if l is odd. Consequently,

$$\zeta(D_p) = \frac{l}{2}(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

Definition 18. *A dipath where the beginning and ending vertices are the same is called an NF-dicycle.*

Remark 1. *Consider $D_c = (\Lambda, \Omega)$ to be an NF-dicycle of a hidden directed route $P_l = (V, C)$, where l denotes an integer with $l \geq 3$. Then:*

(i) $(\varkappa_\Lambda, \varphi_\Lambda, \varpi_\Lambda) = \{(\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k)) : e_k \in \Lambda, \forall k \in \{1, 2, \ldots, l\}\}$.

(ii) $(\varkappa_\Omega, \varphi_\Omega, \varpi_\Omega) = \{(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1})), (\varkappa_\Omega(e_l, e_1), \varphi_\Omega(e_l, e_1), \varpi_\Omega(e_l, e_1)) : (e_k, e_{k+1}), (e_m, e_1) \in \Lambda, \forall k \in \{1, 2, \ldots, l-1\}\}$.

(iii) $(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1})) = (\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k)), (\varkappa_\Lambda(e_{k+1}), \varphi_\Lambda(e_{k+1}), \varpi_\Lambda(e_{k+1}))$ *for all* $k \in \{1, 2, \ldots, l-1\}$.

Theorem 4. *Consider $D_C = (\Lambda, \Omega)$ as an NFD-cycle for a hidden dipath $D_l = (V, C)$, where $l \geq 3$. Then, one of the following requirements must exist:*

(i)

$$\zeta(D_C) = \min\left\{\sum_{r=1}^{l/2}(\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})), \sum_{r=1}^{m/2}(\varkappa_\Lambda(e_{2r}), \varphi_\Lambda(e_{2r}), \varpi_\Lambda(e_{2r}))\right\};$$

(ii)

$$\zeta(D_C) = \min(E \cup F),$$

where
$X = \left\{ \frac{l+1}{2} - \sum_{r=1}^{k} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \sum_{r=\frac{n+3}{2}-k}^{\frac{l+1}{2}} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})) : k \in \left\{0, 1, 2, \ldots, \frac{l-1}{2}\right\} \right\}$,
and
$F = \left\{ \frac{l+1}{2} - \sum_{r=1}^{k} (\varkappa_\Lambda(e_{2r}), \varphi_\Lambda(e_{2r}), \varpi_\Lambda(e_{2r})) + \sum_{r=\frac{l+3}{2}-k}^{\frac{l+1}{2}} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) : k \in \left\{0, 1, 2, \ldots, \frac{l-1}{2}\right\} \right\}$.

Proof. Assume that R = {1, 2, . . . , l}. The arcs $(\varkappa_\Omega(e_k, e_{k+1}), \varphi_\Omega(e_k, e_{k+1}), \varpi_\Omega(e_k, e_{k+1}))$ as well as $(\varkappa_\Omega(e_m, e_1), \varphi_\Omega(e_m, e_1), \varpi_\Omega(e_m, e_1))$ are effective for all $k \in I$. This implies that $(\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k))$ dominates $(\varkappa_\Lambda(e_{k+1}), \varphi_\Lambda(e_{k+1}), \varpi_\Lambda(e_{k+1}))$ for all $k \in \{1, 2, \ldots, l-1\}$ and $(\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l))$ dominates $(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1))$. Case 1: If, for any number r, l is even, that is, $l = 2r$. Following that, the specified collection

$$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \ldots, (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}$$

and

$$\{(\varkappa_\Lambda(e_2), \varphi_\Lambda(e_2), \varpi_\Lambda(e_2)), (\varkappa_\Lambda(e_4), \varphi_\Lambda(e_4), \varpi_\Lambda(e_4)), \ldots, (\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l))\}$$

are both the DS and $\mathcal{MDS}$s of D_c. Take note of that
$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)) + (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)) + \ldots + (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\} =$

$\sum_{r=1}^{m/2} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1}))$ and $\{(\varkappa_\Lambda(e_2), \varphi_\Lambda(e_2), \varpi_\Lambda(e_2)) + (\varkappa_\Lambda(e_4), \varphi_\Lambda(e_4), \varpi_\Lambda(e_4)) + \ldots + (\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l))\} = \sum_{r=1}^{l/2} (\varkappa_\Lambda(e_{2r}), \varphi_\Lambda(e_{2r}), \varpi_\Lambda(e_{2r}))$.
Thus,

$$\zeta(D_C) = \min \left\{ \sum_{r=1}^{l/2} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})), \sum_{r=1}^{l/2} (\varkappa_\Lambda(e_{2r}), \varphi_\Lambda(e_{2r}), \varpi_\Lambda(e_{2r})) \right\}.$$

This proves (1).
Case 2: For every integer r, if $l = 2r - 1$, that is, l, is odd. Next up are the sets
$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \ldots, (\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l))\}$;
$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \ldots, (\varkappa_\Lambda(e_{l-2}), \varphi_\Lambda(e_{l-2}), \varpi_\Lambda(e_{l-2})), (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}$;
$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \ldots, (\varkappa_\Lambda(e_{l-4}), \varphi_\Lambda(e_{l-4}), \varpi_\Lambda(e_{l-4})), (\varkappa_\Lambda(e_{l-3}), \varphi_\Lambda(e_{l-3}) \varpi_\Lambda(e_{l-3})), (\varkappa_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}$;

$$\ldots;$$

$\{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), \ldots, (\varkappa_\Lambda(e_{l-5}), \varphi_\Lambda(e_{l-5}), \varpi_\Lambda(e_{l-5})), (\varkappa_\Lambda(e_{l-3}), \varphi_\Lambda(e_{l-3}) \varpi_\Lambda(e_{l-3})), (\varkappa_\Lambda(e_{l-1}), \varphi_\Lambda(e_{l-1}), \varpi_\Lambda(e_{l-1}))\}$
are a few of the $\mathcal{MDS}$ and $\mathcal{DS}$ of D_c. Take note of that
$(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)) + (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)) + \ldots + (\varkappa_\Lambda(e_l), \varphi_\Lambda(e_l), \varpi_\Lambda(e_l)) =$

$\sum_{r=1}^{\frac{l+1}{2}}(\varkappa_\Lambda(e_{2r-1}),\varphi_\Lambda(e_{2r-1}),\varpi_\Lambda(e_{2r-1}))$.
In general,
$(\varkappa_\Lambda(e_1),\varphi_\Lambda(e_1),\varpi_\Lambda(e_1))+(\varkappa_\Lambda(e_3),\varphi_\Lambda(e_3),\varpi_\Lambda(e_3))+\ldots+(\varkappa_\Lambda(e_{l-5}),\varphi_\Lambda(e_{l-5}),\varpi_\Lambda(e_{l-5}))+(\varkappa_\Lambda(e_{l-3}),\varphi_\Lambda(e_{l-3}),\varpi_\Lambda(e_{l-3}))+(\varkappa_\Lambda(e_{l-1}),\varphi_\Lambda(e_{l-1}),\varpi_\Lambda(e_{l-1}))+(\varkappa_\Lambda(e_l),\varphi_\Lambda(e_l),\varpi_\Lambda(e_l)) = \sum_{r=1}^{\frac{l+1}{2}}(\varkappa_\Lambda(e_{2r}),\varphi_\Lambda(e_{2r}),\varpi_\Lambda(e_{2r})) + \sum_{r=\frac{k+3}{2}}^{\frac{l+1}{2}}(\varkappa_\Lambda(e_{2r-1}),\varphi_\Lambda(e_{2r-1}),\varpi_\Lambda(e_{2r-1}))$.
Let $\frac{k+1}{2}+k=\frac{l+1}{2}.Then,$

$$\frac{l+1}{2}-\sum_{r=1}^{k}(\varkappa_\Lambda(e_{2r}),\varphi_\Lambda(e_{2r}),\varpi_\Lambda(e_{2r}))+\sum_{r=\frac{m+3}{2}-k}^{\frac{l+1}{2}}(\varkappa_\Lambda(e_{2r-1}),\varphi_\Lambda(e_{2r-1}),\varpi_\Lambda(e_{2r-1})),\forall j\in\{0,1,2,\ldots,\frac{l-1}{2}\}.$$

Thus,

$$\zeta(D_c)=\min(E\cup F).$$

This proves demonstrates (2).

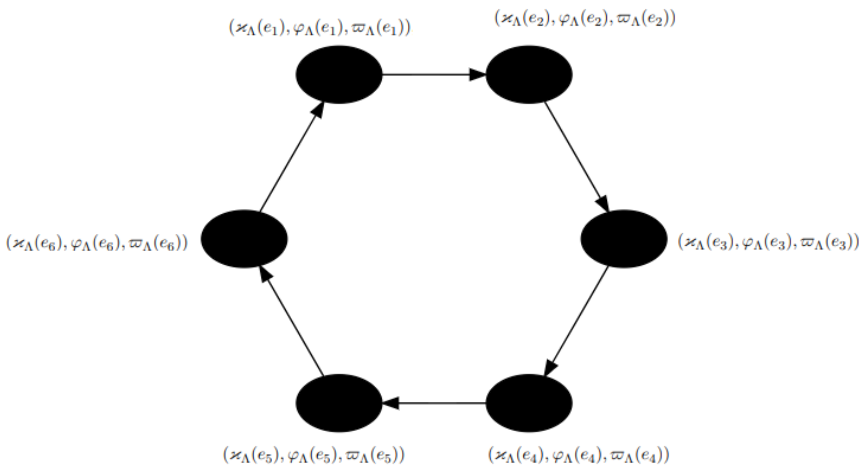

Figure 6: NF Directed cycle with six verticees .

Example 8. *The hidden dicycle $D_6=(V,C)$ has an NF-dicycle $D_c=(\Lambda,\Omega)$. Determine the sets*

$$E=\{(\varkappa_\Lambda(e_1),\varphi_\Lambda(e_1),\varpi_\Lambda(e_1)),(\varkappa_\Lambda(e_3),\varphi_\Lambda(e_3),\varpi_\Lambda(e_3)),(\varkappa_\Lambda(e_5),\varphi_\Lambda(e_5),\varpi_\Lambda(e_5))\}$$

and

$$F=\{(\varkappa_\Lambda(e_2),\varphi_\Lambda(e_2),\varpi_\Lambda(e_2)),(\varkappa_\Lambda(e_4),\varphi_\Lambda(e_4),\varpi_\Lambda(e_4)),(\varkappa_\Lambda(e_6),\varphi_\Lambda(e_6),\varpi_\Lambda(e_6))\}$$

(see Fig. 6). The $\mathcal{MDS}$ of D_c is $\{E,F\}$, and the DN is given by

$$\zeta(D_c)=\min\left\{\sum_{r=1}^{3}(\varkappa_\Lambda(e_{2r-1}),\varphi_\Lambda(e_{2r-1}),\varpi_\Lambda(e_{2r-1})),\sum_{r=1}^{3}(\varkappa_\Lambda(e_{2r}),\varphi_\Lambda(e_{2r}),\varpi_\Lambda(e_{2r}))\right\}.$$

Example 9. *Given a hidden dicycle $D_5 = (V, C)$, let $D_c = (\Lambda, \Omega)$ be an IF dicycle. Consider*

$$\begin{aligned}
E_1 &= \{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\}, \\
E_2 &= \{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_2), \varphi_\Lambda(e_2), \varpi_\Lambda(e_2)), (\varkappa_\Lambda(e_4), \varphi_\Lambda(e_4), \varpi_\Lambda(e_4))\}, \\
E_3 &= \{(\varkappa_\Lambda(e_2), \varphi_\Lambda(e_2), \varpi_\Lambda(e_2)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\}, \\
E_4 &= \{(\varkappa_\Lambda(e_1), \varphi_\Lambda(e_1), \varpi_\Lambda(e_1)), (\varkappa_\Lambda(e_3), \varphi_\Lambda(e_3), \varpi_\Lambda(e_3)), (\varkappa_\Lambda(e_4), \varphi_\Lambda(e_4), \varpi_\Lambda(e_4))\}, \\
E_5 &= \{(\varkappa_\Lambda(e_2), \varphi_\Lambda(e_2), \varpi_\Lambda(e_2)), (\varkappa_\Lambda(e_4), \varphi_\Lambda(e_4), \varpi_\Lambda(e_4)), (\varkappa_\Lambda(e_5), \varphi_\Lambda(e_5), \varpi_\Lambda(e_5))\}.
\end{aligned}$$

Thus, based on Fig. 7, D_c has an $\mathcal{MDS}$ of $\{E_r : r = 1, 2, \ldots, 5\}$ and the DN is $\zeta(D_c) = \min\left\{\sum_{k=1}^{5} (\varkappa_\Lambda(e_k), \varphi_\Lambda(e_k), \varpi_\Lambda(e_k)) \in E_r\right\}$.

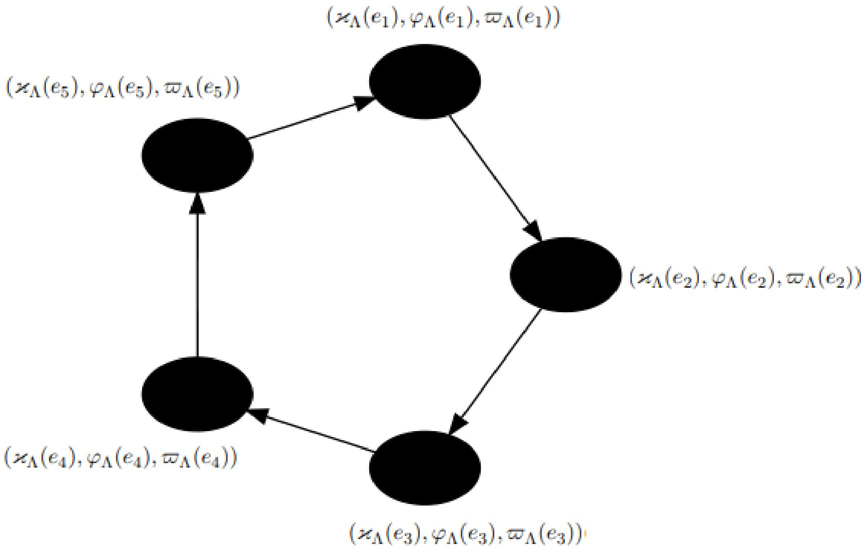

Figure 7: NF Directed cycle with five verticees .

Theorem 5. *If $D_c = (\Lambda, \Omega)$ as a Nf- dicycle of a hidden dicycle $C_p = (V, C)$,where $p \geq 3$. If $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) = (\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f))$ for all $e, f \in V$, then*

$$\left[\frac{l}{2}\right] (\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

Proof. If n is even, by Theorem 4, we have:

$$\frac{l}{2} \sum \gamma(D_c) = \min\left\{\sum_{r=1}^{\frac{l}{2}} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})), \sum_{r=1}^{\frac{l}{2}} (\varkappa_\Lambda(e_{2r}), \varphi_\Lambda(e_{2r}), \varpi_\Lambda(e_{2r}))\right\}.$$

Because $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) = (\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f))$ for all $e, f \in V$, it implies

$$\frac{l}{2}\sum \gamma(D_C) = \left(\frac{l}{2}\right)(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)),$$

using the same reasoning as in Theorem 3.26. Similarly, if m is odd, by Theorem 3.26:

$$\sum_{r=1}^{\frac{l+1}{2}-k} \gamma(D_c) = \sum_{r=1}^{\frac{l+1}{2}-k} (\varkappa_\Lambda(e_{2r-1}), \varphi_\Lambda(e_{2r-1}), \varpi_\Lambda(e_{2r-1})) + \sum_{r=\frac{l+3}{2}-k}^{\frac{l+1}{2}} (\varkappa_\Lambda(e_{2r-2}), \varphi_\Lambda(e_{2r-2}), \varpi_\Lambda(e_{2r-2})),$$

for all $k \in \{0, 1, 2, \ldots, \frac{l-1}{2}\}$. Because $(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)) = (\varkappa_\Lambda(f), \varphi_\Lambda(f), \varpi_\Lambda(f))$ for all $e, f \in V$, we obtain:

$$\frac{l+1}{2}\sum \gamma(D_c) = \left(\frac{l+1}{2}\right)(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

Hence, $\gamma(D_c)$ is either $\left(\frac{l}{2}\right)(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$ if l is even, or $\left(\frac{l+1}{2}\right)(\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e))$ if l is odd. Thus,

$$\zeta(D_c) = \left[\frac{l}{2}\right](\varkappa_\Lambda(e), \varphi_\Lambda(e), \varpi_\Lambda(e)).$$

4. Selection of locations for EV charging stations using dominance in neutrosophic fuzzy directed graphs

Many fuzzy graphs have been used to mimic real-world problems, such as sites for infrastructure projects. Here, we provide a model based on dominance in Neutrosophic Fuzzy Directed Graphs (NFDGs) to determine the best sites for public charging stations for electric vehicles (EVs). For comparison, we first model the scenario using the principles of domination in Fuzzy Directed Graphs (FDGs), and then explore it through the perspective of NFDG dominance. Adopting our suggested NFDG model produces more consistent and trustworthy outcomes than the old FDG strategy.

4.1. Use FDG as a model

If a city planner is assigned the duty of installing public charging stations at different points around the city. The planner is unable to construct charging stations at every place because of financial restrictions. The objective is to determine the bare minimum of charging stations necessary to coverage, guaranteeing that most EV users can readily locate a station.
The prospective sites are depicted as nodes in this FDG model (Fig. 8), and their membership degrees (MD) illustrate the proportion of nearby EV customers who would utilize the charging stations. The key roads that connect these places are shown as directed edges, and the MD of these roads indicates the probability that EV users will

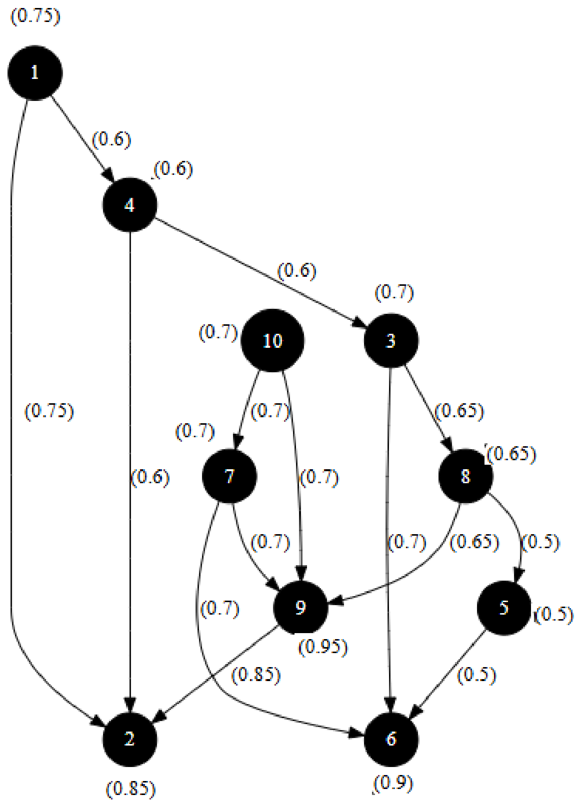

Figure 8: FDG Model .

utilize them.
We ascertain that the set$\{L_2, L_6, L_9\}$ is a $\mathcal{MDS}$ by looking at the effective arcs, such as $(L_1, L_2), (L_1, L_4), (L_9, L_2), (L_1, L_4), (L_7, L_6), (L_8, L_9), (L_7, L_9), (L_10, L_9), (L_8, L_5), (L_5, L_6),$ $(L_3, L_6), (L_3, L_8), (L_10, L_7), (L_4, L_3)$, it shows that 85% of EV users near L_2, 90% near L_6, and 95% nearL_9 are likely to utilize the charging stations. These localities also have significant $\mathcal{MD}$s. These places are ideal for station placement since they are also well-connected to other localities.

4.2. Use the NFDG model

We only took into account the likelihood of usage (liking) in the FDG-based model; we neglected to account for any deterrents (disliking) that can influence EV consumers' decisions. We use the NFDG approach to address this and offer a more in-depth examination. The NFDG is shown in Fig. 9, where the non-membership degrees (NMD) represent the percentage of users who might avoid the charging stations because of things like distance, traffic congestion, or safety concerns, and the MD of the nodes represents the percentage of EV users who are likely to use them. Based on a resident poll, the directed arcs show the likelihood that electric vehicle (EV) users will drive over the roadways that link these places. We determine the dominating set and effective arcs by using the concept of dominance in NFDGs. Using this technique, we discover once more that the set$\{L_2, L_6, L_9\}$ forms an $\mathcal{MDS}$, suggesting ideal charging station sites. Table 2 indicates that these locations are suitable as ideal sites since they have low Falsity-membership (10% forL_2, 5% for L_6, and 3% for L_9) and high Truth-membership (85% for L_2, 90% for L_6, and 95% for L_9).

A more thorough assessment of both positive and negative elements is made possible by the usage of NFDGs, which promotes better decision-making. The city planner can guarantee coverage and customer happiness by choosing these sites, which will promote EV adoption and ensuring the charging network is implemented successfully.

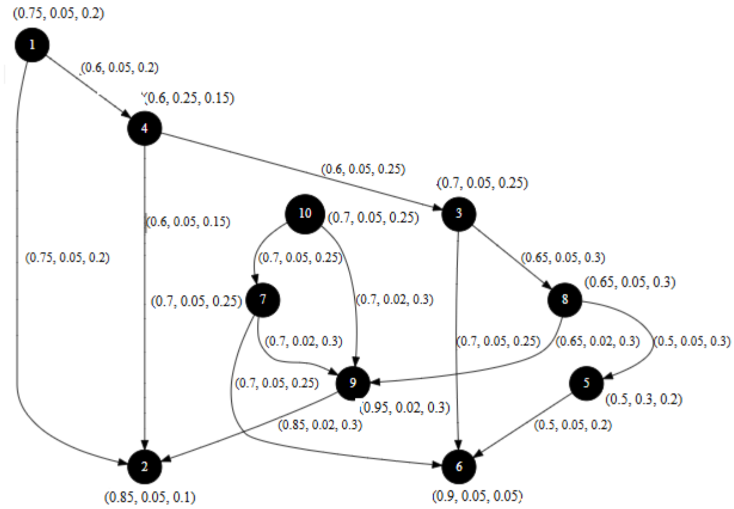

Figure 9: NFDG Model .

Table 2: Membership Values of Locations

Location	Truth-membership	Falsity-membership	Indeterminacy-membership
L_1	0.75	0.2	0.05
L_2	0.85	0.1	0.05
L_3	0.7	0.25	0.05
L_4	0.6	0.15	0.25
L_5	0.5	0.2	0.3
L_6	0.9	0.05	0.05
L_7	0.7	0.25	0.05
L_8	0.65	0.3	0.05
L_9	0.95	0.03	0.02
L_{10}	0.7	0.25	0.05

Pseudo-code for Neutrosophic Fuzzy System

Input: NFS of Locations Λ
Output: MDS

```
void FNFS_MDS() {
        NFS  Λ = getNFSOfLocations();
        int numOfLocations = count(Λ);
        NFS Ω;
        Set DS;

        for (int e = 0; e < numOfLocations; e++) {
                for (int f = 0; f < numOfLocations; f++) {
                        if (Λ(e, f)) {
                                ϰ_Ω(e, f) = min(ϰ_Λ(e), ϰ_Λ(f));
                                φ_Ω(e, f) = max(φ_Λ(e), φ_Λ(f));
                                ϖ_Ω(e, f) = min(ϖ_Λ(e), ϖ_Λ(f));
                        }
                        if (arc_is_effective(e, f, Ω)) {
                                DS.add(e);
                        }
                }
        }
        Set MDS = find_MDS(DS);
}

bool arc_is_effective(int e, int f, NFS Ω) {
        return (ϰ_Ω(e, f) > ϰ_threshold &&
        φ_Ω(e, f) < φ_threshold &&
        ϖ_Ω(e, f) < ϖ_threshold);
}

struct NFS {
        map<pair<int, int>, double> ϰ;
        map<pair<int, int>, double> φ;
        map<pair<int, int>, double> ϖ;

        bool isAdjacent(int e, int f);
};

NFS getNFSOfLocations() {
        NFS Λ;
        return Λ;
}
```

```
int count(NFS Λ) {
        return ϰ_Λ.size();
}
```

5. Comparative analysis

The concept of domination inuzzy graphs (FGs) has proved very important in the solving of real life problems that are characterized by uncertainties. Rosenfeld introduced the concept of Fuzzy graph (FG) where vertices and arcs are associated with a fuzzy membership degree under some conditions. After that, FGs have been used in many area and have productive outcomes and motivated the extension such as fuzzy directed graph (FDGs) where vertex and edge value are fuzzy set. It turns out that simple/certain cases are best addressed with FDFs while these very problems are not very effective when dealing with complexity and uncertainty. This has led to more flexible structures to enhance accuracy giving rise to the neutrosophic fuzzy graphs (NFGs).
Meanwhile, NFSs introduce the truth membership and falsity membership for both vertices and edges of NFGs to overcome the drawback of the FGs and FDGs as the effectiveness and ineffectiveness are balanced. Moreover, this framework is developed by Neutrosophic fuzzy directed graphs (NFDGs) which propose effective arcs for the better accuracy of the right locations for EV charging station sites. For example, FDGs can only show that it has an effectiveness of 95 percent for a location like L_9 , where this single membership value eliminates consideration of ineffectiveness it may introduce bias. However, NFDG approaches use both truth and falsity values, leaded a more fitness estimating for potential EV charging station sites by trade suitable-efficiency pairs, as well as trade unsuitable-efficiency pairs.

In the example above, NFDGs enables one to evaluate various aspects which in turns facilitate a better identification of the best sites to locate EV charging. Thus, NFDGs offer more general and more flexible structure that would expand the ability to accurately model and transform uncertain information in the given domain.

6. Conclusion

This study demonstrates the effectiveness of the NFDGs comparing to the traditional FDGs. The utilization of effective arcs like semi-k, semi-$\varnothing$, and semi-ψ arcs along with the reconceptualization of domination types make the NFDGs a strong platform of analyzing and solving uncertainty based situations. The flexibility of applying NFDGs for modelling higher domination structures like DS or MDS contributes to making of decisions and to real-world use. For example, the usability of NFDGs when solving the problem of locating EV charging stations is clear, as the employment of such algorithms provides better and more effective outcomes compared to straightforward FDGs. These outcomes stress common usage of NFDGs, especially for considering real world problems, where the interpretation of uncertainty is crucial.

The extension of the proposed NFDG framework can also be applicable to various domains such as disaster management, healthcare logistics, and smart transportation systems, where uncertainty prevails over decision-making. Further comparison with other advanced graph models, including intuitionistic fuzzy graphs or bipolar fuzzy graphs could further explicate the advantages and convenient of NFDGs. In addition to that, it would be equally interesting to test the working of this framework in different conditions such as the urban, rural, and the industrial area environments to establish its flexibility and expansion.

Furthermore, identifying improved algorithms for calculation of domination sets and other measures on NFDGs would increase the efficiency of their computation with respect to large-size networks. Extension of the NFDG-based models with the state of the art technologies like AI and machine learning could also pave way for decision making under uncertainty, as and when they are more developed and applied across the implementation arena.

References

[1] N. K. Adichwal, A. A. H. Ahmadini, Y. S. Raghav, R. Singh, and I. Ali. Estimation of general parameters using ancillary information in simple random sampling without replacement. *Journal of King Saud University-Science*, 2021.

[2] A. A. H. Ahmadini, A. Msmali, Z. Mutum, and Y. S. Raghav. The mathematical modeling approach for the wastewater treatment process in saudi arabia during covid-19 pandemic. *Journal of Mathematics*, 2022.

[3] M. Akram, N. Alshehri, B. Davvaz, and A. Ashraf. Bipolar fuzzy digraphs in decision support systems. *J. Mult.-Valued Log. Soft Comput.*, 27, 2016.

[4] M. Akram, N. O. Alshehri, et al. Intuitionistic fuzzy cycles and intuitionistic fuzzy trees. *Sci. World J.*, 2014.

[5] M. Akram, A. Ashraf, M. Sarwar, et al. Novel applications of intuitionistic fuzzy digraphs in decision support systems. *Sci. World J.*, 2014.

[6] M. Akram and W. A. Dudek. Intuitionistic fuzzy hypergraphs with applications. *Inf. Sci.*, 218:182–193, 2013.

[7] M. Akram, A. Sattar, and A. B. Saeid. Competition graphs with complex intuitionistic fuzzy information. *Gran. Comput.*, 7(1):25–47, 2022.

[8] A. Al-Masarwah and G. Ahmad. Subalgebras of type (α, β) based on m-polar fuzzy points in bck/bci-algebras. *AIMS Mathematics*, 5:1035ÔÇô1049, 2020.

[9] A. Al-Masarwah and M. Rajeshwari. Application of complex neutrosophic graphs in hospital infrastructure design. *Mathematics*, 12(5):719, 2024.

[10] Anas Al-Masarwah and Majdoleen Abu Qamar. Some new concepts of fuzzy soft graphs. *Fuzzy Inf. Eng.*, 8(4):427–438, 2016.

[11] Anas Al-Masarwah and Majdoleen Abu Qamar. Certain types of fuzzy soft graphs. *New Math. Nat. Comput.*, 14(2):145–156, 2018.

[12] W. F. Al-Omeri and M. Kaviyarasu. Study on neutrosophic graph with application on earthquake response center in japan. *Symmetry*, 16(6):743, 2024.

[13] Z. Ali. Decision-making techniques based on complex intuitionistic fuzzy power interaction aggregation operators and their applications. *J. Innov. Res. Math. Comput. Sci.*, 1(1):107–125, 2022.

[14] K. Atanassov and G. Gargov. Elements of intuitionistic fuzzy logic. part i. *Fuzzy Sets Syst.*, 95(1):39–52, 1998.

[15] P. Bhattacharya. Some remarks on fuzzy graphs. *Pattern Recognit. Lett.*, 6(5):297–302, 1987.

[16] S. Broumi. Secure dominance in neutrosophic graphs. *Neutrosophic Sets Syst.*, 56(1):7, 2023.

[17] S. Broumi, S. Mohanaselvi, T. Witczak, M. Talea, A. Bakali, and F. Smarandache. Complex fermatean neutrosophic graph and application to decision making. *Decis. Mak. Appl. Manag. Eng.*, 6(1):474–501, 2023.

[18] R. Narmada Devi, M. Meenu, and G. Muthumari. A novel of domination in neutrosophic over graphs. *Neutrosophic Sets Syst.*, 58(1):2, 2023.

[19] E. Enriquez, G. Estrada, C. Loquias, R. J. Bacalso, and L. Ocampo. Domination in fuzzy directed graphs. *Mathematics*, 9(17):2143, 2021.

[20] Y. Fei. Study on neutrosophic graph with application in wireless network. *CAAI Transactions on Intelligence Technology*, 5(4):301–307, 2020.

[21] Takaaki Fujita. Survey of planar and outer planar graphs in fuzzy and neutrosophic graphs. 2024.

[22] R. V. Jaikumar, R. Sundareswaran, and S. Broumi. Integrity and domination integrity in neutrosophic soft graphs. *Neutrosophic Sets Syst.*, 53:165–178, 2023.

[23] C. Jana, H. Garg, and M. Pal. Multi-attribute decision making for power dombi operators under pythagorean fuzzy information with mabac method. *J. Ambient Intell. Humaniz. Comput.*, 14(8):10761–10778, 2023.

[24] M. Kaviyarasu, H. Rashmanlou, S. Kosari, S. Broumi, R. Venitha, M. Rajeshwari, and F. Mofidnakhaei. Circular economy strategies to promote sustainable development using t-neutrosophic fuzzy graph. *Neutrosophic Sets Syst.*, 72(1):9, 2024.

[25] S. U. Khan, A. Nasir, N. Jan, and Z. H. Ma. Graphical analysis of covering and paired domination in the environment of neutrosophic information. *Mathematical Problems in Engineering*, 2021:5518295, 2021.

[26] P. K. Kishore Kumar and S. Lavanya. On fuzzy digraphs. *Int. J. Pure Appl. Math.*, 115(3):599–606, 2017.

[27] T. S. Lakhwani, K. Mohanta, A. Dey, S. P. Mondal, and A. Pal. Some operations on dombi neutrosophic graph. *J. Ambient Intell. Humaniz. Comput.*, pages 1–19, 2022.

[28] R. Mahapatra, S. Samanta, and M. Pal. Applications of edge colouring of fuzzy graphs. *Informatica*, 31(2):313–330, 2020.

[29] J. Malarvizhi and G. Divya. Domination and edge domination in single valued neutrosophic graph. *Infinite Study*, 2021.

[30] A. Meenakshi and J. Senbagamalar. Equitable domination in neutrosophic graph using strong arc. *Neutrosophic Sets Syst.*, 60:59–73, 2023.

[31] S. N. F. Mohamad, R. Hasni, and F. Smarandache. Novel concepts on domina-

tion in neutrosophic incidence graphs with some applications. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 27(5):837–847, 2023.

[32] J. N. Mordeson and P. Chang-Shyh. Operations on fuzzy graphs. *Inf. Sci.*, 79(3ÔÇô4):159–170, 1994.

[33] J. N. Mordeson and P. S. Nair. Successor and source of (fuzzy) finite state machines and (fuzzy) directed graphs. *Inf. Sci.*, 95(1-2):113–124, 1996.

[34] G. Muhiuddina, N. Alenzea, A. Mahboob, and Anas Al-Masarwah. Some new concepts on int-soft ideals in ordered semigroups. *New Mathematics and Natural Computation*, 17(2):267–279, 2021.

[35] M. Mullai and S. Broumi. Dominating energy in neutrosophic graphs. *Int. J. Neutrosoph. Sci.*, 5(1):38–58, 2020.

[36] D. Nithyanandham, F. Augustin, D. R. Micheal, and N. D. Pillai. Energy based bipolar intuitionistic fuzzy digraph decision-making system in selecting covid-19 vaccines. *Appl. Soft Comput.*, 147:110793, 2023.

[37] S. Poulik and G. Ghorai. Estimation of most affected cycles and busiest network route based on complexity function of graph in fuzzy environment. *Artif. Intell. Rev.*, pages 1–18, 2022.

[38] Y. S. Raghav, M. A. Ali, A. Goel, and S. Kumar. The effects of inflation, shortages, and partial backlogs on products that deteriorate over time in response to varied demand. *Brazilian Journal of Business*, 42(4):719, 2023.

[39] Y. S. Raghav, A. Haq, and I. Ali. Multi-objective intuitionistic fuzzy programming under pessimistic and optimistic applications in multivariate stratified sample allocation problems. *PLoS One*, 2023.

[40] Yashpal Singh Raghav. Neutrosophic generalized exponential robust ratio type estimators. *Int. J. Anal. Appl.*, 21:41, 2023.

[41] H. Rashmanlou, S. Samanta, M. Pal, and R. A. Borzooei. Intuitionistic fuzzy graphs with categorical properties. *Fuzzy Inf. Eng.*, 7(3):317–334, 2015.

[42] A. Rosenfeld. Fuzzy graphs, fuzzy sets and their applications to cognitive and decision processes. In *Proceedings of the US-Japan Seminar on Fuzzy Sets and their Applications*, pages 77–95. Academic Press, 1975.

[43] D. Sasikala and B. Divya. A newfangled interpretation on fermatean neutrosophic dombi fuzzy graphs. *Neutrosophic Syst. Appl.*, 7:36–53, 2023.

[44] J. Senbagamalar, A. Meenakshi, T. Kujani, and A. Kanchana. Application of neutrosophic network using efficient domination. pages 499–506, 2023.

[45] V. Senthilkumar. Types of domination in intuitionistic fuzzy graph by strong arc and effective arc. *Bull. Pure Appl. Sci., Math. Stat.*, 37(2):490–498, 2018.

[46] A. Shannon, K. T. Atanassov, et al. A first step to a theory of the intuitionistic fuzzy graphs. In D. Akov, editor, *Proc. of the First Workshop on Fuzzy Based Expert Systems*, pages 59–61, 1994.

[47] S. Singh, A. Barve, K. Muduli, A. Kumar, and S. Luthra. Evaluating roadblocks to implementing a green freight transportation system: an interval-valued intuitionistic fuzzy digraph matrix approach. *IEEE Trans. Eng. Manag.*, 2022.

[48] F. Smarandache, S. N. F. Mohamad, and R. Hasni. Novel concepts on domination

in neutrosophic incidence graphs with some applications. *Neutrosophic Sets Syst.*, 56, 2023.

[49] T. P. Sreelakshmi and K. Uma Samundesvari. Total domination in uniform single valued neutrosophic graph. *Neutrosophic Sets Syst.*, 70(1):305–313, 2024.

[50] M. Sunitha and A. Vijayakumar. Complement of a fuzzy graph. *Indian J. Pure Appl. Math.*, 33(9):1451–1464, 2002.

[51] U. ur Rehman. Selection of database management system by using multi-attribute decision-making approach based on probability complex fuzzy aggregation operators. *J. Innov. Res. Math. Comput. Sci.*, 2(1):1–16, 2023.

[52] N. Vinothkumar and E. Karuppasamy. Complementary domination in single valued neutrosophic graphs. *Materials Today: Proceedings*, 47:2076–2079, 2021.

[53] N. Yaqoob, M. Gulistan, S. Kadry, and H. A. Wahab. Complex intuitionistic fuzzy graphs with application in cellular network provider companies. *Mathematics*, 7(1):35, 2019.

[54] L. A. Zadeh. Fuzzy sets. *Inf. Control*, 8(3):338–353, 1965.

[55] L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning i. *Inf. Sci.*, 8(3):199–249, 1975.