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Quadratic f-hom-ders in Banach Algebra Related to System of Quadratic Functional Equations

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Abstract. Mirzavaziri and Moslehian [13] introduced the concept of f-derivations and Sripattanet *et al.* [18] introduced a quadratic hom-der in Banach algebras. In this paper, we solve the system of quadratic functional equations

$$\begin{cases} f(x+y)+f(x-y)=g(x)+g(y), \\ g\left(\frac{x+y}{2}\right)+g\left(\frac{x-y}{2}\right)=f(x)+f(y). \end{cases}$$

Using Mirzavaziri and Moslehian's idea and Sripattanet *et al.*'s idea, we define a quadratic f-hom-der in Banach algebras, and we investigate the Hyers-Ulam stability of quadratic f-hom-ders in Banach algebras.

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Key Words and Phrases: quadratic f-hom-der; fixed point method; Hyers-Ulam stability; system of quadratic functional equations

1. Introduction

Let $\mathcal{B}$ be a complex Banach algebra and $f:\mathcal{B}\to\mathcal{B}$ be a $\mathbb{C}$-linear mapping. Mirzavaziri and Moslehian [13] introduced the concept of f-derivation $g:\mathcal{B}\to\mathcal{B}$ as follows:

$$g(xy)=f(x)g(y)+g(x)f(y) \tag{1}$$

for all $x,y\in\mathcal{B}$.

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Park *et al.* [16] introduced the concept of hom-derivation on $\mathcal{B}$, i.e., $g : \mathcal{B} \to \mathcal{B}$ is a homomorphism and f satisfies (1) for all $x, y \in \mathcal{B}$. Dehghanian *et al.* introduced the concept of hom-der $g : \mathcal{B} \to \mathcal{B}$ as follows:

$$g(x)g(y) = xg(y) + g(x)y$$

for all $x, y \in \mathcal{B}$. Dehghanian *et al.* [6] introduced and investigated ternary hom-ders in ternary Banach algebras and Kheawborisuk *et al.* [10] defined and studied hom-ders in fuzzy Banach algebras. Recently, Sripattanet *et al.* [18] introduced a quadratic hom-der in Banach algebras $\mathcal{B}$ as follows: A quadratic mapping $D : \mathcal{B} \to \mathcal{B}$ is said to be a quadratic hom-der if it satisfies

$$D(x)D(y) = x^2D(y) + D(x)y^2$$

for all $x, y \in \mathcal{B}$.

In this papere, we introduce the concept of quadartic hom-der in Banach algebras.

Definition 1. *Let $\mathcal{B}$ be a complex Banach algebra and $f : \mathcal{B} \to \mathcal{B}$ be a quadratic mapping. A quadratic mapping $g : \mathcal{B} \to \mathcal{B}$ is called a quadratic f-hom-der if it satisfies*

$$g(x)g(y) = f(x)g(y) + g(x)f(y)$$

for all $x, y \in \mathcal{B}$.

Example 1. *Let $C_0(X)$ be the complex Banach algebra of complex valued continuous functions on a locally compact Hausdorff space X and $g : C_0(X) \to C_0(X)$ be defined by $g(M) = 2M^2$ and $f : C_0(X) \to C_0(X)$ be defined by $f(M) = M^2$. Then f is a quadratic mapping and g is a quadratic f-hom-der.*

We say that an equation is stable if any function satisfying the equation approximately is near to an exact solution of the equation.

The stability analysis of functional equations emanated from a question of Ulam [19], was raised in 1940, about the stability of group homomorphisms and then was extended by Hyers [9]. Recently, results on the so-called Hyers-Ulam stability have comfortabled the stability conditions. Dehghanian and Modarres [3] studied ternary γ-homomorphisms and ternary γ-derivations on ternary semigroups, Dehghanian *et al.* [4] studied ternary 3-derivations on C^*-ternary algebras and Dehghanian and Park [5] studied C^*-ternary 3-homomorphisms on C^*-ternary algebras. Moreover, Senthil Kumar *et al.* [11] investigated modular stabilities of a reciprocal second power functional equation and Bowniya *et al.* [1, 2] obtained the Hyers-Ulam stability results of linear differential equations.

The method provided by Hyers [9] which produces the additive function will be called a direct method. This method is the most significant and strong tool to concerning the stability of different functional equations. That is, the exact solution of the functional equation is explicitly constructed as a limit of a sequence, starting from the given approximate solution [17]. The other significant method is fixed point theorem, that is, the exact solution of the functional equation is explicitly created as a fixed point of some certain mapping [8, 14, 15].

We remember a fixed point alternative theorem.

Theorem 1. *[7] If $(\mathcal{B}, d)$ is a complete generalized metric space and $\mathfrak{I} : \mathcal{B} \to \mathcal{B}$ a strictly contractive mapping, that is,*

$$d(\mathfrak{I}u, \mathfrak{I}v) \leq Ld(u, v)$$

for all $u, v \in \mathcal{B}$ and a Lipschitz constant $L < 1$. Then for each given element $u \in \mathcal{B}$, either

$$d(\mathfrak{I}^n u, \mathfrak{I}^{n+1} u) = +\infty, \qquad \forall n \geq 0,$$

or

$$d(\mathfrak{I}^n u, \mathfrak{I}^{n+1} u) < +\infty, \qquad \forall n \geq n_0,$$

for some positive integer n_0. Furthermore, if the second alternative holds, then

(i) the sequence $(\mathfrak{I}^n u)$ is convergent to a fixed point v^* of $\mathfrak{I}$;

(ii) v^* is the unique fixed point of $\mathfrak{I}$ in the set $V := \{v \in \mathcal{B}, d(\mathfrak{I}^{n_0} u, v) < +\infty\}$;

(iii) $d(v, v^*) \leq \frac{1}{1-L} d(v, \mathfrak{I}v)$ for all $u, v \in V$.

In this paper, we consider the following system of additive functional equations

$$\begin{cases} f(x+y) + f(x-y) = g(x) + g(y), \\ g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) = f(x) + f(y) \end{cases} \tag{2}$$

for all $x, y \in \mathcal{B}$. The aim of the present paper is to solve the system of quadratic functional equations and prove the Hyers-Ulam stability of quadratic f-hom-ders in complex Banach algebras by using the fixed point method.

Throughout this paper, assume that $\mathcal{B}$ is a complex Banach algebra.

2. Stability of system of quadratic functional equations

We solve and investigate the system of quadratic functional equations (2) in complex Banach algebras.

Lemma 1. *Let $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and (2) for all $x, y \in \mathcal{B}$. Then the mappings $f, g : \mathcal{B} \to \mathcal{B}$ are quadratic.*

Proof. Letting $x = y = 0$ in (2), we get

$$f(0) = g(0) = 0.$$

Putting $y = 0$ in (2), we have

$$\begin{aligned} 2f(x) &= g(x), \\ 2g\left(\frac{x}{2}\right) &= f(x) = \frac{1}{2}g(x) \end{aligned} \tag{3}$$

for all $x \in \mathcal{B}$. So

$$f(x+y) + f(x-y) = 2f(x) + 2f(y)$$

for all $x, y \in \mathcal{B}$. Hence the mapping $f : \mathcal{B} \to \mathcal{B}$ is quadratic. Moreover, by (3),

$$g\left(\frac{x}{2}\right) = \frac{1}{4} g(x)$$

and so

$$\frac{1}{4}g(x+y) + \frac{1}{4}g(x-y) = g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) = f(x) + f(y) = \frac{1}{2}g(x) + \frac{1}{2}g(y)$$

for all $x, y \in \mathcal{B}$. Thus

$$g(x+y) + g(x-y) = 2g(x) + 2g(y)$$

for all $x, y \in \mathcal{B}$ and so the mapping $g : \mathcal{B} \to \mathcal{B}$ is quadratic.

Using the fixed point technique, we prove the Hyers-Ulam stability of the system of quadratic functional equations (2) in complex Banach algebras.

Theorem 2. *Suppose that $\Delta : \mathcal{B}^2 \to [0, \infty)$ is a function such that there exists an $L < 1$ with*

$$\Delta\left(\frac{x}{2}, \frac{y}{2}\right) \leq \frac{L}{4} \Delta(x, y) \tag{4}$$

for all $x, y \in \mathcal{B}$. Let $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and

$$\begin{cases} \|f(x+y) + f(x-y) - g(x) - g(y)\| \leq \Delta(x, y), \\ \|g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) - f(x) - f(y)\| \leq \Delta(x, y) \end{cases} \tag{5}$$

for all $x, y \in \mathcal{B}$. Then there exist unique quadratic mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that

$$\|F(x) - f(x)\| \leq \frac{2+L}{4(1-L)} \Delta(x, x), \tag{6}$$

$$\|G(x) - g(x)\| \leq \frac{2+L}{2(1-L)} \Delta(x, x) \tag{7}$$

for all $x \in \mathcal{B}$.

Proof. Putting $x = y = 0$ in (5), we get

$$\begin{cases} \|2f(0) - 2g(0)\| \leq \Delta(0, 0) = 0, \\ \|2g(0) - 2f(0)\| \leq \Delta(0, 0) = 0 \end{cases}$$

and so $f(0) = g(0) = 0$.

Letting $y = x$ in (5), we obtain

$$\begin{cases} \|f(2x) - 2g(x)\| \le \Delta(x,x), \\ \|g(x) - 2f(x)\| \le \Delta(x,x) \end{cases}$$

and so

$$\begin{cases} \left\|g(x) - 4g\left(\frac{x}{2}\right)\right\| \le 2\Delta\left(\frac{x}{2}, \frac{x}{2}\right) + \Delta(x,x) \le \frac{2+L}{2}\Delta(x,x), \\ \left\|f(x) - 4f\left(\frac{x}{2}\right)\right\| \le \Delta\left(\frac{x}{2}, \frac{x}{2}\right) + \frac{1}{2}\Delta(x,x) \le \frac{2+L}{4}\Delta(x,x) \end{cases} \tag{8}$$

for all $x \in \mathcal{B}$.

Let $\Gamma = \{\gamma : \mathcal{B} \to \mathcal{B} : \gamma(0) = 0\}$. We define a generalized metric on Γ as follows: $d : \Gamma \times \Gamma \longrightarrow [0, \infty]$ by

$$d(\delta, \gamma) = \inf\{\mu \in \mathbb{R}_+ : \|\delta(x) - \gamma(x)\| \le \mu\Delta(x,x), \forall x \in \mathcal{B}\},$$

and we consider $\inf \emptyset = +\infty$. Then d is a complete generalized metric on Γ (see [12]).

Now, we define the mapping $\mathcal{J} : (\Gamma, d) \to (\Gamma, d)$ such that

$$\mathcal{J}\delta(x) := 4\delta\left(\frac{x}{2}\right)$$

for all $x \in \mathcal{B}$.

Actually, let $\delta, \gamma \in (\Gamma, d)$ be given such that $d(\delta, \gamma) = \mu$. Then

$$\|\delta(x) - \gamma(x)\| \le \mu\Delta(x,x)$$

for all $x \in \mathcal{B}$. Hence

$$\|\mathcal{J}\delta(x) - \mathcal{J}\gamma(x)\| = \left\|4\delta\left(\frac{x}{2}\right) - 4\gamma\left(\frac{x}{2}\right)\right\| \le 4\mu\Delta\left(\frac{x}{2}, \frac{x}{2}\right) \le L\mu\Delta(x,x)$$

for all $x \in \mathcal{B}$. It follows that $d(\mathcal{J}\delta(x), \mathcal{J}\gamma(x)) \le L\mu$. So

$$d(\mathcal{J}\delta(x), \mathcal{J}\gamma(x)) \le Ld(\delta, \gamma)$$

for all $x \in \mathcal{B}$ and all $\delta, \gamma \in \Gamma$.

It follows from (8) that $d(f, \mathcal{J}f) \le \frac{2+L}{4}$ and $d(g, \mathcal{J}g) \le \frac{2+L}{2}$.

Using the fixed point alternative we deduce the existence of a unique fixed point of $\mathcal{J}$ and a unique fixed point of $\mathcal{J}$, that is, the existence of mappings $F, G : \mathcal{B} \to \mathcal{B}$, respectively, such that

$$F(x) = 4F\left(\frac{x}{2}\right), \quad G(x) = 4G\left(\frac{x}{2}\right)$$

with the following property: there exist $\mu, \eta \in (0, \infty)$ satisfying

$$\|f(x) - F(x)\| \le \mu\Delta(x,x), \quad \|g(x) - G(x)\| \le \eta\Delta(x,x)$$

for all $x \in \mathcal{B}$.

Since $\lim_{n\to+\infty} d(\mathcal{J}^n f, F) = 0$ and $\lim_{n\to+\infty} d(\mathcal{J}^n g, G) = 0$,

$$\lim_{n\to+\infty} 4^n f\left(\frac{x}{2^n}\right) = F(x), \qquad \lim_{n\to+\infty} 4^n g\left(\frac{x}{2^n}\right) = G(x)$$

for all $x \in \mathcal{B}$.

Next, $d(f, F) \le \frac{1}{1-L} d(f, \mathcal{J}f)$ and $d(g, G) \le \frac{1}{1-L} d(g, \mathcal{J}g)$ which imply

$$\|f(x) - F(x)\| \le \frac{2+L}{4(1-L)}\Delta(x,x), \qquad \|g(x) - G(x)\| \le \frac{2+L}{2(1-L)}\Delta(x,x)$$

for all $x \in \mathcal{B}$.

Using (4) and (5), we conclude that

$$\begin{aligned}
&\|F(x+y) + F(x-y) - G(x) - G(y)\| \\
&= \lim_{n\to+\infty} 4^n \left\| f\left(\frac{x+y}{2^n}\right) + f\left(\frac{x-y}{2^n}\right) - g\left(\frac{x}{2^n}\right) - g\left(\frac{y}{2^n}\right) \right\| \\
&\le \lim_{n\to+\infty} 4^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \le \lim_{n\to+\infty} L^n \Delta(x,y) = 0
\end{aligned}$$

and

$$\begin{aligned}
&\left\|G\left(\frac{x+y}{2}\right) + G\left(\frac{x-y}{2}\right) - F(x) - F(y)\right\| \\
&= \lim_{n\to+\infty} 4^n \left\| g\left(\frac{x+y}{2\cdot 2^n}\right) + g\left(\frac{x-y}{2\cdot 2^n}\right) - f\left(\frac{x}{2^n}\right) - f\left(\frac{y}{2^n}\right) \right\| \\
&\le \lim_{n\to+\infty} 4^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \le \lim_{n\to+\infty} L^n \Delta(x,y) = 0
\end{aligned}$$

for all $x, y \in \mathcal{B}$, since $L < 1$. Hence

$$\begin{cases} F(x+y) + F(x-y) = G(x) + G(y), \\ G\left(\frac{x+y}{2}\right) + G\left(\frac{x-y}{2}\right) = F(x) + F(y) \end{cases}$$

for all $x, y \in \mathcal{B}$. Therefore by Lemma 1, the mappings $F, G : \mathcal{B} \to \mathcal{B}$ are quadratic.

Corollary 1. *Let p and q be nonnegative real numbers with $p + q > 4$ and $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and*

$$\begin{cases} \|f(x+y) + f(x-y) - g(x) - g(y)\| \le \|x\|^p \|y\|^q, \\ \\ \|g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) - f(x) - f(y)\| \le \|x\|^p \|y\|^q \end{cases}$$

for all $x, y \in \mathcal{B}$. Then there exist unique quadratic mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that

$$\begin{aligned}
\|F(x) - f(x)\| &\le \frac{2^{p+q} + 8}{2(2^{p+q} - 16)} \|x\|^{p+q}, \\
\|G(x) - g(x)\| &\le \frac{2^{p+q} + 8}{2^{p+q} - 16} \|x\|^{p+q}
\end{aligned}$$

for all $x \in \mathcal{B}$.

Proof. The proof follows from Theorem 2 by taking $L = \frac{2^4}{2^{p+q}}$ and $\Delta(x, y) = \|x\|^p\|y\|^q$ for all $x, y \in \mathcal{B}$.

Corollary 2. *Let p and θ be nonnegative real numbers with $p > 4$ and $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and*

$$\begin{cases} \|f(x+y) + f(x-y) - g(x) - g(y)\| \le \theta(\|x\|^p + \|y\|^p), \\ \|g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) - f(x) - f(y)\| \le \theta(\|x\|^p + \|y\|^p) \end{cases}$$

for all $x, y \in \mathcal{B}$. Then there exist unique quadratic mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that

$$\|F(x) - f(x)\| \le \frac{2^p + 2}{2^p - 4}\theta\|x\|^p,$$
$$\|G(x) - g(x)\| \le \frac{2(2^p + 2)}{2^p - 4}\theta\|x\|^p$$

for all $x \in \mathcal{B}$.

Proof. The proof follows from Theorem 2 by taking $L = \frac{4}{2^p}$ and $\Delta(x, y) = \theta(\|x\|^p + \|y\|^p)$ for all $x, y \in \mathcal{B}$.

3. Stability of quadratic F-hom-ders in Banach algebras

In this section, by using the fixed point technique, we prove the Hyers-Ulam stability of quadratic F-hom-ders in complex Banach algebras.

Theorem 3. *Suppose that $\Delta : \mathcal{B}^2 \to [0, \infty)$ is a function such that there exists an $L < 1$ with*

$$\Delta(x, y) \le \frac{L}{16}\Delta(2x, 2y) \tag{9}$$

for all $x, y \in \mathcal{B}$. Let $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and

$$\begin{cases} \|f(x+y) + f(x-y) - g(x) - g(y)\| \le \Delta(x, y), \\ \|g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) - f(x) - f(y)\| \le \Delta(x, y) \end{cases} \tag{10}$$

and

$$\|g(x)g(y) - f(x)g(y) - g(x)f(y)\| \le \Delta(x, y) \tag{11}$$

for all $x, y \in \mathcal{B}$. Then there exist unique quadratic mappings $F, G : \mathcal{B} \to \mathcal{B}$ satisfying (6) and (7) and G is a quadratic F-hom-der.

Proof. Since

$$\Delta(x,y) \leq \frac{L}{16}\Delta(2x,2y) \leq \frac{L}{4}\Delta(2x,2y)$$

for all $x, y \in \mathcal{B}$, by Theorem 2, there exist unique mappings $F, G : \mathcal{B} \to \mathcal{B}$ satisfying (6) and (7) which are given by

$$\lim_{n\to+\infty} 4^n f\left(\frac{x}{2^n}\right) = F(x), \qquad \lim_{n\to+\infty} 4^n g\left(\frac{x}{2^n}\right) = G(x)$$

for all $x \in \mathcal{B}$.

It follows from (11) that

$$\begin{aligned}
&\|G(x)G(y) - F(x)G(y) - G(x)F(y)\| \\
&= \lim_{n\to+\infty} 16^n \left\| g\left(\frac{x}{2^n}\right) g\left(\frac{y}{2^n}\right) - f\left(\frac{x}{2^n}\right) g\left(\frac{y}{2^n}\right) - g\left(\frac{x}{2^n}\right) f\left(\frac{y}{2^n}\right)\right\| \\
&\leq \lim_{n\to+\infty} 16^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \\
&\leq \lim_{n\to+\infty} L^n \Delta(x,y) = 0
\end{aligned}$$

for all $x, y \in \mathcal{B}$. So

$$G(x)G(y) = F(x)G(y) + G(x)F(y)$$

for all $x, y \in \mathcal{B}$. Thus the quadratic mapping G is a quadratic F-hom-der.

Corollary 3. *Let p and q be nonnegative real numbers with $p + q > 4$ and $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and*

$$\begin{cases} \|f(x+y) + f(x-y) - g(x) - g(y)\| \leq \|x\|^p \|y\|^q, \\ \|g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) - f(x) - f(y)\| \leq \|x\|^p \|y\|^q \end{cases}$$

and

$$\|g(xy) - f(x)g(y) - g(x)f(y)\| \leq \|x\|^p \|y\|^q$$

for all $x, y \in \mathcal{B}$. Then there exist unique quadratic mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that G is a quadratic F-hom-der and

$$\begin{aligned}
\|F(x) - f(x)\| &\leq \frac{2^{p+q} + 8}{2(2^{p+q} - 16)} \|x\|^{p+q}, \\
\|G(x) - g(x)\| &\leq \frac{2^{p+q} + 8}{2^{p+q} - 16} \|x\|^{p+q}
\end{aligned}$$

for all $x \in \mathcal{B}$.

Proof. The proof follows from Theorem 3 by taking $\Delta(x,y) = \|x\|^p\|y\|^q$ for all $x, y \in \mathcal{B}$. Choosing $L = 2^{4-p-q}$, we obtain the desired result.

Corollary 4. *Let p and θ be nonnegative real numbers with $p > 4$ and $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying $g(0) = 0$ and*

$$\begin{cases} \|f(x+y) + f(x-y) - g(x) - g(y)\| \le \theta(\|x\|^p + \|y\|^q), \\ \|g\left(\frac{x+y}{2}\right) + g\left(\frac{x-y}{2}\right) - f(x) - f(y)\| \le \theta(\|x\|^p + \|y\|^q) \end{cases}$$

and

$$\|g(xy) - f(x)g(y) - g(x)f(y)\| \le \theta(\|x\|^p + \|y\|^q)$$

for all $x, y \in \mathcal{B}$. Then there exist unique quadratic mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that G is a quadratic F-hom-der and

$$\|F(x) - f(x)\| \le \frac{2^p + 2}{2^p - 4}\theta\|x\|^p,$$
$$\|G(x) - g(x)\| \le \frac{2(2^p + 2)}{2^p - 4}\theta\|x\|^p$$

for all $x \in \mathcal{B}$.

Proof. The proof follows from Theorem 3 by taking $\Delta(x,y) = \theta(\|x\|^p + \|y\|^q)$ for all $x, y \in \mathcal{B}$. Choosing $L = 2^{4-p}$, we obtain the desired result.

4. Conclusion and future work

We solved the system of quadratic functional equations (2) and we defined quadratic f-hom-ders in Banach algebras and investigated the Hyers-Ulam stability of quadratic f-hom-ders in Banach algebras. We will define cubic f-hom-ders and quartic f-hom-ders in Banach algebras, fuzzy Banach algebras and non-Archimedean Banach algebras and investigate the Hyers-Ulam stability of them.

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