EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 1, Article Number 5785
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Quasi Contractions and Fixed Point Theorems in the Context of Neutrosophic Fuzzy Metric Spaces

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Abstract. Fixed point theory has garnered significant interest due to its applicability across various scientific disciplines. In this manuscript, we present fixed point theorems pertaining to neutrosophic fuzzy quasi-contractions, situated within the sophisticated framework of neutrosophic fuzzy metric spaces. Furthermore, we establish several fixed point results pertinent to this field of research.

2020 Mathematics Subject Classifications: 37C25, 55M20, 26E50, 46S40

Key Words and Phrases: Fixed point theory, Neutrosophic fuzzy metric spaces, Banach contraction principle, quasi-contractions, linear contractions

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1. Introduction

The Banach fixed-point theorem [6], often referred to as the Banach contraction principle, represents a cornerstone in mathematical analysis, especially within the realm of metric spaces. This theorem guarantees both the existence and uniqueness of fixed points for certain self-maps, provided specific conditions are met in metric spaces. Essentially, the Banach fixed-point theorem offers a comprehensive framework for Picard's method of successive approximations and has found extensive application across various branches of mathematics. Introduced by Stefan Banach (1892–1945) in 1922, this theorem has inspired a multitude of mathematicians to explore numerous extensions and generalizations in diverse mathematical fields, as indicated by the references in [2, 8–10, 22, 23, 25, 29, 34–36].

In [30], the authors present the concept of $\breve{\alpha}, \breve{\eta}$ proximal contractive mappings. Subsequently, they establish a theorem regarding the best proximity point for this class of mappings within the context of a fuzzy Banach space, demonstrating significant results related to best proximity points for these contractions. In the work referenced as [33], the authors established the existence of coincidence and best proximity points for fuzzy $(\alpha-\eta)$- and fuzzy $(\beta-\psi)$-generalized proximal contractions within the context of b-fuzzy

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DOI: https://doi.org/10.29020/nybg.ejpam.v18i1.5785

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metric spaces. In the work referenced as [31], the authors defined fuzzy normed spaces and presented the concepts of the best proximity point and $\tilde{\alpha}-$proximal admissibility within this framework. Additionally, they introduced the concept of $\tilde{\alpha}, \tilde{\psi}$-proximal contractive mappings. A theorem regarding the best proximity point for these types of mappings in fuzzy normed spaces was subsequently established. In reference [32], the authors established the existence and uniqueness of the best proximity point for nonself-mappings within a fuzzy normed space.

The notion of Fuzzy Sets (FSs), first introduced by Zadeh [39], has significantly influenced a wide array of scientific fields since its emergence. While this framework is highly pertinent to practical applications, it has not consistently offered satisfactory solutions to various problems over the years. As a result, there has been a renewed interest in research focused on resolving these challenges. In this context, Atanassov [4] proposed Intuitionistic Fuzzy Sets (IFSs) as a strategy to tackle such issues. Furthermore, the Neutrosophic Set (NS), created by Smarandache [37], serves as a sophisticated extension of traditional set theory. Other important generalizations include interval-valued FS [38], interval-valued IFS [5].

Neutrosophic sets exhibit a diverse range of applications across multiple fields. For instance, Barbosa and Smarandache [7] proposed the Neutrosophic One-Round Zero-Knowledge Proof protocol (N-1-R) ZKP, which enhances the One-Round (1-R) ZKP framework by incorporating Neutrosophic numbers. Furthermore, the authors in [1] provide an in-depth examination of effective and optimally appropriate solutions related to scalar optimization problems. They also derive the Kuhn-Tucker conditions relevant to both efficiency and optimal efficiency. To gain a more comprehensive insight into the applications of neutrosophic sets and their extensive uses, it is recommended to consult the literature referenced in [3, 11, 14, 16–20] and associated sources.

2. Preliminary

In this framework, the interval $]0-, 1+[$ is characterized as a non-standard unit interval. Within this context, non-standard finite numbers are articulated as $(1+) = 1 + \epsilon$, where "1" represents the standard component and ϵ denotes the non-standard element. In a similar manner, $(0-) = 0 - \epsilon$, with "0" indicating the standard component and 'ϵ" as the non-standard element. The numbers 0 and 1 can be interpreted as non-standard values that are infinitesimally small yet less than 0 or infinitesimally small yet greater than 1, respectively, and these values are encompassed within the non-standard unit interval $]0-, 1+[$.

Definition 1. *[39] In relation to a universal set U, a fuzzy set $\mathcal{X}$ is defined by the notation $\mathcal{X} = \{<\xi, \mu_{\mathcal{X}}(\xi)>: 0 \leq \mu_{\mathcal{X}}(\xi) \leq 1, \xi \in U\}$. In this context, $\mu_{\mathcal{X}} : \mathcal{X} \to [0, 1]$, and $\mu_{\mathcal{X}}(\xi)$ represents the degree of membership of the element ξ within the fuzzy set $\mathcal{X}$.*

Definition 2. *[37] A neutrosophic set V relative to a universal set U is defined as $V = \{<\xi, (T_N(\xi), I_N(\xi), F_N(\xi))>: \xi \in U, T_N(\xi), I_N(\xi), F_N(\xi) \in\ ]0-, 1+[\ \}$. In this context, $T_N(\xi)$, $I_N(\xi)$, and $F_N(\xi)$ represent the membership degrees of truth, indeterminacy, and* *falsity for an element ξ within the set V, respectively, while $]0-, 1+[$ signifies a non-standard unit interval.*

Definition 3. *[13] A neutrosophic fuzzy set D within a universal set U is characterized as follows: $D = \{< x, (\mu_D(\xi), T_D(\xi,\mu), I_D(\xi,\mu), F(\xi,\mu)) >: \xi \in U, \mu_D(\xi) \in [0,1],$ $T_D(\xi,\mu), I_D(\xi,\mu), F(\xi,\mu) \in]0-, 1+[\ \}$. In this framework, the membership degree $\mu_D(\xi)$ is represented by three distinct components: the truth membership grade $T_D(\xi,\mu)$, the indeterminacy membership grade $I_D(\xi,\mu)$, and the falsity membership grade $F(\xi,\mu)$. The notation $]0-, 1+[$ signifies a nonstandard unit interval.*

Triangular norms (TN), first proposed by Menger [26] (see also [24]), represent a crucial concept in the realm of mathematical analysis. Menger's pioneering methodology utilized probability distributions to evaluate the distance between two elements in a given space, thereby transcending the conventional dependence on numerical values. This approach enables the generalization of the triangle inequality within metric spaces via the implementation of triangular norms. In contrast, triangular conorms (TC) act as dual counterparts to t-norms. Both TN and TC play vital roles in fuzzy operations, particularly in relation to intersections and unions.

In this manuscript, the notation $\mathbb{R}^+$ is used to represent the interval $= [0, +\infty)$, while I denotes the unit interval $[0, 1]$.

Definition 4. *[26]*

Consider an operation $\odot : I \times I \to I$. This operation is classified as continuous t-norm (TN) if it meets the following criteria: for any elements $\varpi, \varpi', t, t' \in I$.

(i) $\varpi \odot 1 = \varpi$,

(ii) If $\varpi \leq \varpi'$ *and* $t \leq t'$, *than* $\varpi \odot t \leq \varpi' \odot t'$,

(iii) $\odot$ *is continuous,*

(iv) $\odot$ *is commutative and associate.*

Definition 5. *[26]*

Consider an operation $\oplus : I \times I \to I$. This operation is classified as continuous t-conorm (TC) if it meets the following criteria: for all elements $\varpi, \varpi', t, t' \in I$.

(i) $\varpi \oplus 0 = \varpi$,

(ii) If $\varpi \leq \varpi'$ *and* $t \leq t'$, *than* $\varpi \oplus t \leq \varpi' \oplus t'$,

(iii) $\oplus$ *is continuous,*

(iv) $\oplus$ *is commutative and associate.*

Definition 6. *[24] A 6-tuple $(\mathcal{X}, \mathcal{H}, \mathcal{K}, \mathcal{L}, \odot, \oplus)$ is referred to as a Neutrophic Metric Space (NMS) if the set $\mathcal{X}$ is a non-empty arbitrary collection, $\odot$ signifies a continuous t-norm, $\oplus$ indicates a continuous t-conorm, and the elements $\mathcal{H}, \mathcal{K}$, and $\mathcal{L}$ are three fuzzy sets established on the Cartesian product $\mathcal{X}^2 \times (0, +\infty)$. These components must satisfy the following specific conditions for all elements $\xi, \omega, c \in \mathcal{X}$ and for all positive real numbers λ, ρ.*

(i) $0 \leq \mathcal{H}(\xi, \omega, \lambda) \leq 1$, $0 \leq \mathcal{K}(\xi, \omega, \lambda) \leq 1$, $0 \leq \mathcal{L}(\xi, \omega, \lambda) \leq 1$,

(ii) $0 \leq \mathcal{H}(\xi, \omega, \lambda) + \mathcal{K}(\xi, \omega, \lambda) + \mathcal{L}(\xi, \omega, \lambda) \leq 3$,

(iii) $\mathcal{H}(\xi, \omega, \lambda) = 1$, *for* $\lambda > 0$ *iff* $\xi = \omega$

(iv) $\mathcal{H}(\xi, \omega, \lambda) = H(\omega, \xi, \lambda)$, *for* $\lambda > 0$

(v) $\mathcal{H}(\xi, \omega, \lambda) \odot \mathcal{H}(\omega, c, \rho) \leq \mathcal{H}(\xi, c, \lambda + \rho)$

(vi) $\mathcal{H}(\xi, \omega, \cdot) : \mathbb{R}^+ \to I$ *is continuous*

(vii) $\lim_{\lambda \to +\infty} \mathcal{H}(\xi, \omega, \lambda) = 1$

(viii) $\mathcal{K}(\xi, \omega, \lambda) = 0$ iff $\xi = \omega$

(ix) $\mathcal{K}(\xi, \omega, \lambda) = \mathcal{K}(\omega, \xi, \lambda)$,

(x) $\mathcal{K}(\xi, \omega, \lambda) \oplus \mathcal{K}(\omega, c, \rho) \geq \mathcal{K}(\xi, c, \lambda + \rho)$,

(xi) $\mathcal{K}(\xi, \omega, \cdot) : \mathbb{R}^+ \to I$ *is continuous*

(xii) $\lim_{\lambda \to +\infty} \mathcal{K}(\xi, \omega, \lambda) = 0$

(xiii) $\mathcal{L}(\xi, \omega, \lambda) = 0$, *for* $\lambda > 0$ *iff* $\xi = \omega$

(xiv) $\mathcal{L}(\xi, \omega, \lambda) = \mathcal{L}(\omega, \xi, \lambda)$,

(xv) $\mathcal{L}(\xi, \omega, \lambda) \oplus \mathcal{L}(\omega, c, \rho) \geq S(\xi, c, \lambda + \rho)$,

(xvi) $\mathcal{L}(\xi, \omega, \cdot) : \mathbb{R} \to I$ *is continuous*

(xvii) $\lim_{\lambda \to +\infty} \mathcal{L}(\xi, \omega, \lambda) = 0$

(xviii) *If* $\lambda \leq 0$, *then* $\mathcal{H}(\xi, \omega, \lambda) = 0$, $\mathcal{K}(\xi, \omega, \lambda) = \mathcal{L}(\xi, \omega, \lambda) = 1$

The functions $\mathcal{H}(\xi, \omega, \lambda)$, $\mathcal{K}(\xi, \omega, \lambda)$, and $\mathcal{L}(\xi, \omega, \lambda)$ represent the degrees of nearness, neutralness, and non-nearness between the elements ξ and ω in relation to the parameter λ, respectively.

Recently, Ghosh et al. [15] presented the notion of neutrosophic fuzzy metric spaces and examined various topological characteristics associated with this concept.

Definition 7. *[15] A 7-tuple* $(\mathcal{X}, \mathcal{H}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \odot, \oplus)$ *is defined as a Neutrophic Fuzzy Metric Space (NFMS) if* $\mathcal{X}$ *represents an arbitrary set,* $\odot$ *denotes a continuous t-norm,* $\oplus$ *signifies a continuous t-conorm, and the fuzzy sets* $\mathcal{H}, \mathcal{J}, \mathcal{K}$*, and* $\mathcal{L}$ *are defined on* $\mathcal{X}^2 \times (0, +\infty)$*. These sets must satisfy specific conditions for all* $\xi, \omega, c \in \mathcal{X}$ *and for* $\lambda, \rho > 0$*.*

(i) $0 \leq \mathcal{H}(\xi, \omega, \lambda) \leq 1$, $0 \leq \mathcal{J}(\xi, \omega, \lambda) \leq 1$, $0 \leq \mathcal{K}(\xi, \omega, \lambda) \leq 1$, $0 \leq \mathcal{L}(\xi, \omega, \lambda) \leq 1$,

(ii) $0 \leq \mathcal{H}(\xi, \omega, \lambda) + \mathcal{J}(\xi, \omega, \lambda) + \mathcal{K}(\xi, \omega, \lambda) + \mathcal{L}(\xi, \omega, \lambda) \leq 4$,

(iii) $\mathcal{H}(\xi, \omega, \lambda) = 1$,

(iv) $\mathcal{H}(\xi, \omega, \lambda) = H(\omega, \xi, \lambda)$,

(v) $\mathcal{H}(\xi, \omega, \lambda) \odot \mathcal{H}(\omega, c, \rho) \leq \mathcal{H}(\xi, c, \lambda + \rho)$, *for* $\rho, \lambda > 0$

(vi) $\mathcal{H}(\xi, \omega, \cdot) : \mathbb{R} \to I$ *is continuous*

(vii) $\lim_{\lambda \to +\infty} \mathcal{H}(\xi, \omega, \lambda) = 1$

(viii) $\mathcal{J}(\xi, \omega, \lambda) = 1$, *iff* $\xi = \omega$

(ix) $\mathcal{J}(\xi, \omega, \lambda) = \mathcal{J}(\omega, \xi, \lambda)$, *for* $\lambda > 0$

(x) $\mathcal{J}(\xi, \omega, \lambda) \odot \mathcal{J}(\omega, c, \rho) \leq \mathcal{J}(\xi, c, \lambda + \rho)$,

(xi) $\mathcal{J}(\xi, \omega, \cdot) : \mathbb{R} \to I$ *is continuous*

(xii) $\lim_{\lambda \to +\infty} \mathcal{J}(\xi, \omega, \lambda) = 1$

(xiii) $\mathcal{K}(\xi, \omega, \lambda) = 0$, *iff* $\xi = \omega$

(xiv) $\mathcal{K}(\xi, \omega, \lambda) = \mathcal{K}(\omega, \xi, \lambda)$,

(xv) $\mathcal{K}(\xi, \omega, \lambda) \oplus \mathcal{K}(\omega, c, \rho) \geq \mathcal{K}(\xi, c, \lambda + \rho)$,

(xvi) $\mathcal{K}(\xi, \omega, \cdot) : \mathbb{R} \to I$ *is continuous*

(xvii) $\lim_{\lambda \to +\infty} \mathcal{K}(\xi, \omega, \lambda) = 0$

(xviii) $\mathcal{L}(\xi, \omega, \lambda) = 0$, *iff* $\xi = \omega$

(xix) $\mathcal{L}(\xi, \omega, \lambda) = \mathcal{L}(\omega, \xi, \lambda)$,

(xx) $\mathcal{L}(\xi, \omega, \lambda) \oplus \mathcal{L}(\omega, c, \rho) \geq \mathcal{L}(\xi, c, \lambda + \rho)$,

(xxi) $\mathcal{L}(\xi, \omega, \cdot) : \mathbb{R} \to I$ *is continuous*

(xxii) $\lim_{\lambda \to +\infty} \mathcal{L}(\xi, \omega, \lambda) = 0$

(xxiii) *If* $\lambda \leq 0$*, then* $\mathcal{H}(\xi, \omega, \lambda) = 0$, $\mathcal{K}(\xi, \omega, \lambda) = \mathcal{L}(\xi, \omega, \lambda) = 1$

In this context, $\mathcal{H}(\xi,\omega,\lambda)$ indicates the certainty that the distance between ξ and ω is less than λ. Meanwhile, $\mathcal{J}(\xi,\omega,\lambda)$ signifies the level of proximity, $\mathcal{K}(\xi,\omega,\lambda)$ represents the degree of neutrality, and $\mathcal{L}(\xi,\omega,\lambda)$ reflects the extent of non-proximity between ξ and ω in relation to λ.

The convergence, Cauchyness, completeness are given as follows.

Definition 8. *[15] Let (ξ_n) be a sequence in a NFMS $(\mathcal{X},\mathcal{H},\mathcal{J},\mathcal{K},\mathcal{L},\odot,\oplus)$. Then*

(i) *(ξ_n) converges to $\xi\in\mathcal{X}$ iff for a given $\epsilon\in(0,1)$, $\lambda>0$ there is $n_0\in\mathbb{N}$ such that for each $n\geq n_0$*

$$\mathcal{H}(\xi_n,\xi,\lambda)>1-\epsilon,\ \mathcal{J}(\xi_n,\xi,\lambda)>1-\epsilon,\ \mathcal{K}(\xi_n,\xi,\lambda)<\epsilon,\ \mathcal{L}(\xi_n,\xi,\lambda)<\epsilon$$

i.e.,

$$\lim_{n\to+\infty}\mathcal{H}(\xi_n,\xi,\lambda)=1,\ \lim_{n\to+\infty}\mathcal{J}(\xi_n,\xi,\lambda)=1,$$
$$\lim_{n\to+\infty}\mathcal{K}(\xi_n,\xi,\lambda)=0,\ \lim_{n\to+\infty}\mathcal{L}(\xi_n,\xi,\lambda)=0$$

(ii) *(ξ_n) is called Cauchy iff for a given $\epsilon\in(0,1)$, $\lambda>0$ there is $n_0\in\mathbb{N}$ such that for each $n,m\geq n_0$*

$$\mathcal{H}(\xi_n,\xi_m,\lambda)>1-\epsilon,\ \mathcal{J}(\xi_n,\xi_m,\lambda)>1-\epsilon,\ \mathcal{K}(\xi_n,\xi_m,\lambda)<\epsilon,\ \mathcal{L}(\xi_n,\xi_m,\lambda)<\epsilon$$

i.e.,

$$\lim_{n.m\to+\infty}\mathcal{H}(\xi_n,\xi_m,\lambda)=1,\ \lim_{n.m\to+\infty}\mathcal{J}(\xi_n,\xi_m,\lambda)=1,$$
$$\lim_{n,m\to+\infty}\mathcal{K}(\xi_n,\xi_m,\lambda)=0,\ \lim_{n,m\to+\infty}\mathcal{L}(\xi_n,\xi_m,\lambda)=0$$

(iii) *$(\mathcal{X},\mathcal{H},\mathcal{J},\mathcal{K},\mathcal{L},\odot,\oplus)$ is called complete if each Cauchy sequence is convergent to an element in $\mathcal{X}$.*

Quasi contractions were first introduced by L.B. Ciric in 1974 within the framework of metric spaces [12]. Ciric established both the existence and uniqueness of fixed points for quasi contractions. Subsequently, numerous researchers have explored quasi contractions in various distance settings, as one can see the studies presented in [21, 27, 28] and references therein.

3. Main Result

In the following sections, we will first present a valuable lemma that is integral to our primary research. Subsequently, we will introduce our contractions within the framework of neutrosophic fuzzy metric spaces. We will then demonstrate that these contractions have a unique fixed point under certain conditions, and we will explore several consequential results derived from our main findings.

We start with the following helpful lemma.

Lemma 1. *Let* $(\mathcal{X}, \mathcal{H}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \odot, \oplus)$ *be a NFMS. Then*

(i) $\mathcal{H}(\xi, \omega, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-decreasing*

(ii) $\mathcal{J}(\xi, \omega, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-decreasing*

(iii) $\mathcal{K}(\xi, \omega, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-increasing*

(iv) $\mathcal{L}(\xi, \omega, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-increasing*

Proof. (1) Let $\lambda_1, \lambda_2 > 0$, with $\lambda_1 > \lambda_2$. Then, there is $\delta > 0$ such that $\lambda_1 = \lambda_2 + \delta$. From (5), we get

$$\begin{aligned} \mathcal{H}(\xi, \omega, \lambda_1) &= \mathcal{H}(\xi, \omega, \lambda_2 + \delta) \\ &\geq \mathcal{H}(\xi, \omega, \lambda_2) \odot \mathcal{H}(\omega, \omega, \delta) \\ &= \mathcal{H}(\xi, \omega, \lambda_2). \end{aligned}$$

The proofs for (2),(3) and (4) are identical to that of (1).

Through this context we need the following notations.
If $\mathcal{T} : \mathcal{X}^2 \times \mathbb{R} \to \mathbb{R}$, and f is a self map on $\mathcal{X}$, then

(i)
$$\begin{aligned} \mathscr{M}^1_{\mathcal{T}}(\xi, \omega, \lambda) &= \max\left\{\frac{1}{\mathcal{T}(\xi, \omega, \lambda)} - 1, \frac{1}{\mathcal{T}(\xi, f\xi, \lambda)} - 1, \frac{1}{\mathcal{T}(\omega, f\omega, \lambda)} - 1, \right. \\ &\qquad \left. \frac{1}{\mathcal{T}(\xi, f\omega, \lambda)} - 1, \frac{1}{\mathcal{T}(f\xi, \omega, \lambda)} - 1\right\}, \end{aligned}$$

(ii) $\mathscr{M}^2_{\mathcal{T}}(\xi, \omega, \lambda) = \max\left\{\mathcal{T}(\xi, \omega, \lambda), \mathcal{T}(\xi, f\xi, \lambda), \mathcal{T}(\omega, f\omega, \lambda), \mathcal{T}(\xi, f\omega, \lambda), \mathcal{T}(f\xi, \omega, \lambda)\right\}.$

Additionally, we establish the following notation to define the concept of need:

(i) $\delta^1_{\mathcal{T}}(\xi, f, \lambda) = \max_{i,j \in \mathbb{N}} \left\{\frac{1}{\mathcal{T}(f^i\xi, f^j\xi, \lambda)} - 1\right\},$

(ii) $\delta^2_{\mathcal{T}}(\xi, f, \lambda) = \max_{i,j \in \mathbb{N}} \left\{\mathcal{T}(f^i\xi, f^j\xi, \lambda)\right\}.$

We will now provide the definition of a neutrosophic fuzzy quasi-contractions.

Definition 9. *Let* $(\mathcal{X}, \mathcal{H}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \odot, \oplus)$ *represent a neutrosophic fuzzy metric space (NFMS). A function* $f : \mathcal{X} \to \mathcal{X}$ *is defined as a neutrosophic fuzzy quasi-contraction if, for every pair of elements* $\xi, \omega \in \mathcal{X}$ *and for all* $\lambda > 0$*, the following condition holds for some* $k \in [0, 1)$*:*

$$\frac{1}{\mathcal{H}(f\xi, f\omega, \lambda)} - 1 \leq k\ \mathscr{M}^1_{\mathcal{H}}(\xi, \omega, \lambda),$$

$$\frac{1}{\mathcal{J}(f\xi, f\omega, \lambda)} - 1 \leq k\ \mathscr{M}^1_{\mathcal{J}}(\xi, \omega, \lambda),$$

$$\mathcal{K}(f\xi, f\omega, \lambda) \le k\ \mathscr{M}^2_{\mathcal{K}}(\xi, \omega, \lambda),$$

and

$$\mathcal{L}(f\xi, f\omega, \lambda) \le k\ \mathscr{M}^2_{\mathcal{L}}(\xi, \omega, \lambda).$$

Theorem 1. *Let $(\mathcal{X}, \mathcal{H}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \odot, \oplus)$ be a complete NFMS, Suppose that $f : \mathcal{X} \to \mathcal{X}$ is neutrosophic fuzzy quasi-contraction. Furthermore, consider that one of the following statements holds true.*

(*) f is continuous,

(**) The fuzzy sets $\mathcal{H}, \mathcal{J}, \mathcal{K}$, and $\mathcal{L}$ exhibit continuity with respect to their first two coordinates.

Consequently, the function f possesses a unique fixed point.

Proof. Let $\xi_0 \in \mathcal{X}$ represent any arbitrary element. We consider the sequence (ξ_n) characterized by the relation $\xi_n = f^n(\xi_0)$ for all $n \ge 0$. By Definition 9 we have for all $i, j \in \mathbb{N}$

$$\begin{aligned}\frac{1}{\mathcal{H}(f^{n+i}\xi_0, f^{n+j}\xi_0, \lambda)} - 1 \quad &\le k\ \max\Big\{\frac{1}{\mathcal{H}(f^{n+i-1}\xi_0, f^{n+j-1}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{H}(f^{n+i-1}\xi_0, f^{n+i}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{H}(f^{n+j-1}\xi_0, f^{n+j}\xi_0, \lambda)} - 1,\\ &\qquad \frac{1}{\mathcal{H}(f^{n+i-1}\xi_0, f^{n+j}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{H}(f^{n+j-1}\xi_0, f^{n+i}\xi_0, \lambda)} - 1\Big\},\end{aligned}$$

$$\begin{aligned}\frac{1}{\mathcal{J}(f^{n+i}\xi_0, f^{n+j}\xi_0, \lambda)} - 1 \quad &\le k\ \max\Big\{\frac{1}{\mathcal{J}(f^{n+i-1}\xi_0, f^{n+j-1}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{J}(f^{n+i-1}\xi_0, f^{n+i}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{J}(f^{n+j-1}\xi_0, f^{n+j}\xi_0, \lambda)} - 1,\\ &\qquad \frac{1}{\mathcal{J}(f^{n+i-1}\xi_0, f^{n+j}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{J}(f^{n+j-1}\xi_0, f^{n+i}\xi_0, \lambda)} - 1\Big\},\end{aligned}$$

$$\begin{aligned}\mathcal{K}(f^{n+i}\xi_0, f^{n+j}\xi_0, \lambda) \quad &\le k\ \max\big\{\mathcal{K}(f^{n+i-1}\xi_0, f^{n+j-1}\xi_0, \lambda), \mathcal{K}(f^{n+i-1}\xi_0, f^{n+i}\xi_0, \lambda), \mathcal{K}(f^{n+j-1}\xi_0, f^{n+j}\xi_0, \lambda),\\ &\qquad \mathcal{K}(f^{n+i-1}\xi_0, f^{n+j}\xi_0, \lambda), \mathcal{K}(f^{n+j-1}\xi_0, f^{n+i}\xi_0, \lambda)\big\},\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}(f^{n+i}\xi_0, f^{n+j}\xi_0, \lambda) \quad &\le k\ \max\big\{\mathcal{L}(f^{n+i-1}\xi_0, f^{n+j-1}\xi_0, \lambda), \mathcal{L}(f^{n+i-1}\xi_0, f^{n+i}\xi_0, \lambda), \mathcal{L}(f^{n+j-1}\xi_0, f^{n+j}\xi_0, \lambda),\\ &\qquad \mathcal{L}(f^{n+i-1}\xi_0, f^{n+j}\xi_0, \lambda), \mathcal{L}(f^{n+j-1}\xi_0, f^{n+i}\xi_0, \lambda)\big\}.\end{aligned}$$

Thus, we conclude

$$\begin{aligned}\delta_{\mathcal{H}}(f^n\xi_0, f, \lambda) &\le k\ \delta^1_{\mathcal{H}}(f^{n-1}\xi_0, f, \lambda),\\ \delta_{\mathcal{J}}(f^n\xi_0, f, \lambda) &\le k\ \delta^1_{\mathcal{J}}(f^{n-1}\xi_0, f, \lambda),\\ \sigma_{\mathcal{K}}(f^n\xi_0, f, \lambda) &\le k\ \delta^2_{\mathcal{K}}(f^{n-1}\xi_0, f, \lambda),\end{aligned}$$

and

$$\sigma_{\mathcal{L}}(f^n\xi_0, f, \lambda) \le k\ \delta^2_{\mathcal{L}}(f^{n-1}\xi_0, f, \lambda).$$

Hence, we conclude that for each $n \geq 1$

$$\begin{aligned}
\delta_{\mathcal{H}}(f^n\xi_0, f, \lambda) &\leq k^n \ \delta^1_{\mathcal{H}}(\xi_0, f, \lambda),\\
\delta_{\mathcal{J}}(f^n\xi_0, f, \lambda) &\leq k^n \ \delta^1_{\mathcal{J}}(\xi_0, f, \lambda),\\
\sigma_{\mathcal{K}}(f^n\xi_0, f, \lambda) &\leq k^n \ \delta^2_{\mathcal{K}}(\xi_0, f, \lambda),
\end{aligned}$$

and

$$\sigma_{\mathcal{L}}(f^n\xi_0, f, \lambda) \leq k^n \ \delta^2_{\mathcal{L}}(\xi_0, f, \lambda).$$

The above inequalities, gives

$$\begin{aligned}
\frac{1}{\mathcal{H}(f^n\xi_0, f^{n+m}\xi_0, \lambda)} - 1 &\leq \delta_{\mathcal{H}}(f^n\xi_0, f, \lambda) \leq k^n \ \delta^1_{\mathcal{H}}(\xi_0, f, \lambda),\\
\frac{1}{\mathcal{J}(f^n\xi_0, f^{n+m}\xi_0, \lambda)} - 1 &\leq \delta_{\mathcal{J}}(f^n\xi_0, f, \lambda) \leq k^n \ \delta^1_{\mathcal{J}}(\xi_0, f, \lambda),\\
\mathcal{K}(f^n\xi_0, f^{n+m}\xi_0, \lambda) &\leq \sigma_{\mathcal{K}}(f^n\xi_0, f, \lambda) \leq k^n \ \delta^2_{\mathcal{K}}(\xi_0, f, \lambda),
\end{aligned}$$

and

$$\mathcal{L}(f^n\xi_0, f^{n+m}\xi_0, \lambda) \leq \sigma_{\mathcal{L}}(f^n\xi_0, f, \lambda) \leq k^n \ \delta^2_{\mathcal{L}}(\xi_0, f, \lambda).$$

By considering the limit as both n and m tend to infinity, we obtain that

$$\begin{aligned}
\lim_{n,m\to+\infty} \mathcal{H}(f^n\xi_0, f^{n+m}\xi_0, \lambda) &= 1,\\
\lim_{n,m\to+\infty} \mathcal{J}(f^n\xi_0, f^{n+m}\xi_0, \lambda) &= 1,\\
\lim_{n,m\to+\infty} \mathcal{K}(f^n\xi_0, f^{n+m}\xi_0, \lambda) &= 0,\\
\lim_{n,m\to+\infty} \mathcal{L}(f^n\xi_0, f^{n+m}\xi_0, \lambda) &= 0,
\end{aligned}$$

Consequently, the sequence $(f^n\xi_0)$ qualifies as a Cauchy sequence, which implies the existence of an element $u \in \mathcal{X}$ such that $f^n\xi_0$ converges to u.
If (*) holds, (i.e., f is continuous), then $f^{n+1}\xi_0 = ff^n\xi_0 \to fu$ and so, $u = fu$.
If (**) holds, then Definition 9 implies that

$$\begin{aligned}
\frac{1}{\mathcal{H}(fu, f^{n+1}\xi_0, \lambda)} - 1 \quad \leq k \ \max\Big\{ & \frac{1}{\mathcal{H}(u, f^n\xi_0, \lambda)} - 1, \frac{1}{\mathcal{H}(u, fu, \lambda)} - 1, \frac{1}{\mathcal{H}(f^n\xi_0, f^{n+1}\xi_0, \lambda)} - 1,\\
& \frac{1}{\mathcal{H}(u, f^{n+1}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{H}(f^n\xi_0, fu, \lambda)} - 1 \Big\},
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\mathcal{J}(fu, f^{n+1}\xi_0, \lambda)} - 1 \quad \leq k \ \max\Big\{ & \frac{1}{\mathcal{J}(u, f^n\xi_0, \lambda)} - 1, \frac{1}{\mathcal{J}(u, fu, \lambda)} - 1, \frac{1}{\mathcal{J}(f^n\xi_0, f^{n+1}\xi_0, \lambda)} - 1,\\
& \frac{1}{\mathcal{J}(u, f^{n+1}\xi_0, \lambda)} - 1, \frac{1}{\mathcal{J}(f^n\xi_0, fu, \lambda)} - 1 \Big\},
\end{aligned}$$

$$\begin{aligned}\mathcal{K}(fu, f^{n+1}\xi_0, \lambda) \quad &\leq k \ \max\big\{\mathcal{K}(u, f^{n}\xi_0, \lambda), \mathcal{K}(u, fu, \lambda), \mathcal{K}(f^{n}\xi_0, f^{n+1}\xi_0, \lambda),\\ &\qquad \mathcal{K}(u, f^{n+1}\xi_0, \lambda), \mathcal{K}(f^{n}\xi_0, fu, \lambda)\ \big\},\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}(fu, f^{n+1}\xi_0, \lambda) \quad &\leq k \ \max\big\{\mathcal{L}(u, f^{n}\xi_0, \lambda), \mathcal{L}(u, fu, \lambda), \mathcal{L}(f^{n}\xi_0, f^{n+1}\xi_0, \lambda),\\ &\qquad \mathcal{L}(u, f^{n+1}\xi_0, \lambda), \mathcal{L}(f^{n}\xi_0, fu, \lambda)\ \big\}.\end{aligned}$$

By evaluating the limit, we obtain

$$(1-k)\ \left(\frac{1}{\mathcal{H}(fu, u, \lambda)} - 1\right) \leq 0,$$

$$(1-k)\ \left(\frac{1}{\mathcal{J}(fu, u, \lambda)} - 1\right) \leq 0,$$

$$(1-k)\ \mathcal{K}(fu, u, \lambda) \leq 0,$$

$$(1-k)\ \mathcal{L}(fu, u, \lambda) \leq 0.$$

Hence $u = fu$.

Let $y \in \mathcal{X}$ such that $y = fy$. If $u \neq y$, then according to Definition 9, it can be inferred that.

$$\begin{aligned}\tfrac{1}{\mathcal{H}(u,y,\lambda)} - 1 = \tfrac{1}{\mathcal{H}(fu,fy,\lambda)} - 1 \quad &\leq k \ \max\Big\{\tfrac{1}{\mathcal{H}(u,y,\lambda)} - 1, \tfrac{1}{\mathcal{H}(u,u,\lambda)} - 1, \tfrac{1}{\mathcal{H}(y,y,\lambda)} - 1,\\ &\qquad \tfrac{1}{\mathcal{H}(u,y,\lambda)} - 1, \tfrac{1}{\mathcal{H}(y,u,\lambda)} - 1\ \Big\}\\ &= k\ \left(\tfrac{1}{\mathcal{H}(u,y,\lambda)} - 1\right),\end{aligned}$$

$$\begin{aligned}\tfrac{1}{\mathcal{J}(u,y,\lambda)} - 1 = \tfrac{1}{\mathcal{J}(fu,fy,\lambda)} - 1 \quad &\leq k \ \max\Big\{\tfrac{1}{\mathcal{J}(u,y,\lambda)} - 1, \tfrac{1}{\mathcal{J}(u,u,\lambda)} - 1, \tfrac{1}{\mathcal{J}(y,y,\lambda)} - 1,\\ &\qquad \tfrac{1}{\mathcal{J}(u,y,\lambda)} - 1, \tfrac{1}{\mathcal{J}(y,u,\lambda)} - 1\ \Big\}\\ &= k\ \left(\tfrac{1}{\mathcal{J}(u,y,\lambda)} - 1\right),\end{aligned}$$

$$\begin{aligned}\mathcal{K}(u, y, \lambda) = \mathcal{K}(fu, fy, \lambda) \quad &\leq k \ \max\{\mathcal{K}(u, y, \lambda), \mathcal{K}(u, u, \lambda), \mathcal{K}(y, y, \lambda),\\ &\qquad \mathcal{K}(u, y, \lambda), \mathcal{K}(y, u, \lambda)\ \}\\ &= k\ \mathcal{K}(y, u, \lambda),\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}(u, y, \lambda) = \mathcal{L}(fu, fy, \lambda) \quad &\leq k \ \max\{\mathcal{L}(u, y, \lambda), \mathcal{L}(u, u, \lambda), \mathcal{L}(y, y, \lambda),\\ &\qquad \mathcal{L}(u, y, \lambda), \mathcal{L}(y, u, \lambda)\ \}\\ &= k\ \mathcal{L}(y, u, \lambda).\end{aligned}$$

So,

$$(1-k)\ \left(\frac{1}{\mathcal{H}(u,y,\lambda)}-1\right)\leq 0,$$
$$(1-k)\ \left(\frac{1}{\mathcal{J}(u,y,\lambda)}-1\right)\leq 0,$$
$$(1-k)\ \mathcal{K}(u,y,\lambda)\leq 0,$$

and

$$(1-k)\ \mathcal{L}(u,y,\lambda)\leq 0.$$

Which implies $u=y$.

Corollary 1. *Let $(\mathcal{X},\mathcal{H},\mathcal{J},\mathcal{K},\mathcal{L},\odot,\oplus)$ be a complete NFMS, Suppose that there is $k\in[0,1)$ such that $f:\mathcal{X}\to\mathcal{X}$ satisfies the following for each $\xi,\omega\in\mathcal{X}$ and each $\lambda>0$, we have:*

$$\frac{1}{\mathcal{H}(f\xi,f\omega,\lambda)}-1\leq k\ \left(\frac{1}{\mathcal{H}(\xi,\omega,\lambda)}-1\right),$$
$$\frac{1}{\mathcal{J}(f\xi,f\omega,\lambda)}-1\leq k\ \left(\frac{1}{\mathcal{J}(\xi,\omega,\lambda)}-1\right),$$
$$\mathcal{K}(f\xi,f\omega,\lambda)\leq k\ \mathcal{K}(\xi,\omega,\lambda),$$

and

$$\mathcal{L}(f\xi,f\omega,\lambda)\leq k\ \mathcal{L}(\xi,\omega,\lambda).$$

Furthermore, consider that one of the following statements holds true.

(i) f is continuous,

(ii) The fuzzy sets $\mathcal{H},\mathcal{J},\mathcal{K}$, and $\mathcal{L}$ exhibit continuity with respect to their first two coordinates.

Consequently, the function f has a unique fixed point.

Corollary 2. *Let $(\mathcal{X},\mathcal{H},\mathcal{J},\mathcal{K},\mathcal{L},\odot,\oplus)$ be a complete NFMS, Suppose that there is $k\in[0,1)$ such that $f:\mathcal{X}\to\mathcal{X}$ satisfies the following for each $\xi,\omega\in\mathcal{X}$ and each $\lambda>0$, we have:*

$$\frac{1}{\mathcal{H}(f\xi,f\omega,\lambda)}-1\leq\frac{k}{2}\ \left(\frac{1}{\mathcal{H}(\xi,f\xi,\lambda)}-1+\frac{1}{\mathcal{H}(\omega,f\omega,\lambda)}-1\right),$$
$$\frac{1}{\mathcal{J}(f\xi,f\omega,\lambda)}-1\leq\frac{k}{2}\ \left(\frac{1}{\mathcal{J}(\xi,f\xi,\lambda)}-1+\frac{1}{\mathcal{J}(\omega,f\omega,\lambda)}-1\right),$$
$$\mathcal{K}(f\xi,f\omega,\lambda)\leq\frac{k}{2}\ (\mathcal{K}(\xi,f\xi,\lambda)+\mathcal{K}(\omega,f\omega,\lambda)),$$

and

$$\mathcal{L}(f\xi, f\omega, \lambda) \leq \frac{k}{2}\ \mathcal{L}(\xi, f\xi, \lambda) + \mathcal{L}(\omega, f\omega, \lambda)).$$

Furthermore, consider that one of the following statements holds true.

(i) f *is continuous,*

(ii) The fuzzy sets $\mathcal{H}, \mathcal{J}, \mathcal{K}$, *and* $\mathcal{L}$ *exhibit continuity with respect to their first two coordinates.*

Consequently, the function f *has a unique fixed point.*

Corollary 3. *Let* $(\mathcal{X}, \mathcal{H}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \odot, \oplus)$ *be a complete NFMS, Suppose that there is* $k \in [0, 1)$ *such that* $f : \mathcal{X} \to \mathcal{X}$ *satisfies the following for each* $\xi, \omega \in \mathcal{X}$ *and each* $\lambda > 0$, *we have:*

$$\frac{1}{\mathcal{H}(f\xi, f\omega, \lambda)} - 1 \leq \frac{k}{2}\ \left(\frac{1}{\mathcal{H}(\xi, f\omega, \lambda)} - 1 + \frac{1}{\mathcal{H}(\xi, f\omega, \lambda)} - 1\right),$$

$$\frac{1}{\mathcal{J}(f\xi, f\omega, \lambda)} - 1 \leq \frac{k}{2}\ \left(\frac{1}{\mathcal{J}(\xi, f\omega, \lambda)} - 1 + \frac{1}{\mathcal{J}(\xi, f\omega, \lambda)} - 1\right),$$

$$\mathcal{K}(f\xi, f\omega, \lambda) \leq \frac{k}{2}\ (\mathcal{K}(\xi, f\omega, \lambda) + \mathcal{K}(\omega, f\xi, \lambda)),$$

and

$$\mathcal{L}(f\xi, f\omega, \lambda) \leq \frac{k}{2}\ (\mathcal{L}(\xi, f\omega, \lambda) + \mathcal{L}(\omega, f\xi, \lambda)).$$

Furthermore, consider that one of the following statements holds true.

(i) f *is continuous,*

(ii) The fuzzy sets $\mathcal{H}, \mathcal{J}, \mathcal{K}$, *and* $\mathcal{L}$ *exhibit continuity with respect to their first two coordinates.*

Consequently, the function f *has a unique fixed point.*

4. Conclusion

The theory of fixed points stands as a cornerstone in both applied and pure mathematics, boasting a diverse array of applications across numerous fields. In this exploration, we present several fixed point theorems pertaining to neutrosophic fuzzy quasi-contractions, elegantly situated within the sophisticated framework of neutrosophic fuzzy metric spaces. Furthermore, we unveil a multitude of fixed point results that are pertinent to this intriguing domain of study. Future research could extend these findings to various distance spaces, such as neutrosophic 2-metric spaces, and explore potential applications related to our work.

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