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## Variance-ratio Tests Robust to a Break in Drift

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**Abstract.** We consider a simple random walk process which exhibits a deterministic break in its drift term: for instance, from positive to negative. We demonstrate both theoretically and by simulation that when the standard variance ratio test is applied to this process, the phenomenon of spurious rejections of the random walk hypothesis can occur. We further propose a modified version of the variance ratio test to avoid such a problem. Finally, we discuss some implications of this finding on the previously revealed empirical evidence against the random walk hypothesis for exchange rates.

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### 1. Introduction

The random walk model plays an important role in many areas of finance and economics. One example is the efficient market hypothesis that if a capital market (or foreign exchange market) is efficient, then all relevant information is fully reflected in the current level of stock prices or exchange rates, and unexpected news is the sole determinant for causing any changes. Therefore, changes of stock prices or exchange rates are not predictable, and the efficient market hypothesis is usually modelled by postulating that the time series of interest follows the random walk process.

Since the seminal work of [4] and [10, 11], the standard variance ratio (VR) test or its improved modifications have been widely used to test the random walk hypothesis. For

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examples of applications, see [6, 19, 3, 25] and the references cited in [20, 21]. A well-known problem with the VR test, namely that the VR statistic is biased and right-skewed in finite samples, is addressed and a solution is provided in a series of theoretical papers such as those by [5, 21, 14, 2].

On the other hand, a growing body of research, especially in the unit root literature, has revealed that many time series data are better characterised when single or multiple breaks are taken into account in modelling the time series. A single break in trend is permitted in [12, 13, 16, 15, 8] when testing the unit root hypothesis, while a single break in variance is considered in [7] and [1].

Despite the growing empirical evidence and theoretical development for structural changes in time series, not much attention has been paid to the effect of such structural breaks on testing the random walk hypothesis. In this paper, we focus on the standard VR test originally proposed by [10, 11] and we allow for a break in the drift term of the time series under investigation. We demonstrate both theoretically and by simulation that the standard VR test displays the phenomenon of spurious rejections of the random walk hypothesis in the presence of such a break. As an example, we discuss some implications of this finding on the previously revealed empirical evidence against the random walk hypothesis for exchange rates.

The paper is organized as follows. In Section 2, we show analytically that the variance ratio test statistics of [10] do not tend to follow the standard normal distribution when there is a break in drift and we demonstrate that in such circumstances, the probability of rejecting the random walk hypothesis approaches one even though the null of the random walk hypothesis is true. The same phenomenon is analysed in finite samples by simulation in Section 3 and we propose a modified variance ratio test in Section 4. Section 5 applies both the standard variance ratio tests of [10] and our new test to the exchange rates of four currencies. We have found that the standard tests strongly reject the random walk hypothesis while our tests do not. Our empirical findings strongly indicate that rejecting the random walk hypothesis by the standard variance ratio tests might have been induced by failing to incorporate structural breaks into the testing procedure. Finally, Section 6 provides a summary of the paper.

## 2. Spurious Rejections of the Random Walk Hypothesis by Variance Ratio Tests

We consider the following process:

$$X_t = \mu_t + X_{t-1} + \varepsilon_t \quad ; \quad t = 1, 2, \dots, T, \quad (1)$$

or

$$r_t = \Delta X_t = \mu_t + \varepsilon_t \quad ; \quad \Delta X_t \equiv X_t - X_{t-1}. \quad (2)$$

The drift term is specified as

$$\mu_t = \mu_1 1(t \leq [\tau T]) + \mu_2 1([\tau T] < t),$$

where  $1(\cdot)$  is the indicator function and  $[\tau T]$  is the integer part of  $\tau T$  and  $\tau \in (0, 1)$ . Whenever there is no confusion, we will use  $\tau T$  in place of  $[\tau T]$  from now on. The disturbance

term  $\varepsilon_t$  is assumed to satisfy the condition that  $E(\varepsilon_t) = 0$  and  $E(\varepsilon_t^2) = \sigma^2$ . The null hypothesis is that  $r_t$  is not serially correlated at all leads and lags; that is,  $E(\varepsilon_t \varepsilon_{t-s}) = 0$  for  $t \neq s$ , which we call the random walk hypothesis. We also assume for convenience that  $X_0$  is observed. The specification of the process in (1) and (2) for testing the random walk hypothesis has been used in previous research. The only difference is that we allow a deterministic break in the drift term.

Using the fact that, under the random walk hypothesis, the variance of the  $q$ -period return,  $X_t - X_{t-q}$ , is equal to  $q$  times the variance of the one-period return,  $X_t - X_{t-1}$ , [10] developed the variance ratio test as follows. The  $q$ -period return,  $r_t(q)$  is defined using overlapping observations:

$$r_t(q) = \sum_{i=0}^{q-1} r_{t-i} = X_t - X_{t-q}.$$

Variance ratio statistics could also be based on non-overlapping returns, but, as shown by [10], using overlapping observations results in a more efficient test. Hence, we only consider variance ratio statistics computed using overlapping observations. The test statistic is based on

$$M(q) = \frac{\hat{\sigma}_q^2}{\hat{\sigma}^2} - 1,$$

where

$$\begin{aligned} \hat{\sigma}^2 &= \frac{1}{T-1} \sum_{t=1}^T (r_t - \hat{\mu})^2, & \hat{\mu} &= \frac{1}{T} \sum_{t=1}^T r_t \\ \hat{\sigma}_q^2 &= \frac{T}{q(T-q+1)(T-q)} \sum_{t=q}^T \{r_t(q) - q\hat{\mu}\}^2. \end{aligned}$$

Lo and MacKinlay [10] proved, under the random walk hypothesis without a break in drift ( $\mu_1 = \mu_2$ ), that (i)  $M(q) \xrightarrow{p} 0$  and (ii) the standardized variance ratio statistic  $z(q) = T^{1/2} \left( \frac{2(2q-1)(q-1)}{3q} \right)^{-1/2} M(q)$  converges in distribution to  $N(0, 1)$ . Hence, given a specific value of  $q$ , appropriate critical values can be chosen from  $N(0, 1)$  to ensure the asymptotically correct size of the test. However, the following theorem demonstrates that it is not possible to control the test size if there is a break in drift ( $\mu_1 \neq \mu_2$ ).

**Theorem 1.** *Suppose that  $X_t$  is generated by (1) under the random walk hypothesis. Then,*

$$M(q) \xrightarrow{p} \frac{q\delta^2\tau(1-\tau)}{1+\delta^2\tau(1-\tau)},$$

where  $\delta = \sigma^{-1}|\mu_1 - \mu_2|$ .

*Proof.* Recalling that  $M_r(q) = \hat{\sigma}^{-2}\hat{\sigma}_q^2 - 1$ , we first examine the probability limit of  $\hat{\sigma}^2$  :

$$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^T (r_t - \hat{\mu})^2 + o_p(1)$$

$$\begin{aligned}
 &= \tau\mu_1^2 + (1 - \tau)\mu_2^2 + T^{-1} \sum_{t=1}^T \varepsilon_t - \widehat{\mu}^2 \\
 &\xrightarrow{p} \sigma^2\{1 + \delta^2\tau(1 - \tau)\}
 \end{aligned} \tag{3}$$

using the fact that

$$\widehat{\mu} \xrightarrow{p} \tau\mu_1 + (1 - \tau)\mu_2.$$

Next, we turn to the variance estimator based on  $q$ -period returns

$$\begin{aligned}
 \widehat{\sigma}_q^2 &= (qT)^{-1} \sum_{t=q}^T (r_t(q) - q\widehat{\mu})^2 + o_p(1) \\
 &= q^{-1} \sum_{i=0}^{q-1} T^{-1} \sum_{t=q}^T (r_{t-i} - \widehat{\mu})^2 + q^{-1} \sum_{i=0}^{q-1} \sum_{j=0(\neq i)}^{q-1} T^{-1} \sum_{t=q}^T (r_{t-i} - \widehat{\mu})(r_{t-j} - \widehat{\mu}).
 \end{aligned}$$

It is straightforward to show that

$$\begin{aligned}
 &T^{-1} \sum_{t=q}^T (r_{t-i} - \widehat{\mu})^2 \xrightarrow{p} \sigma^2\{1 + \delta^2\tau(1 - \tau)\} \\
 &T^{-1} \sum_{t=q}^T (r_{t-i} - \widehat{\mu})(r_{t-j} - \widehat{\mu}) \xrightarrow{p} \sigma^2\delta^2\tau(1 - \tau).
 \end{aligned}$$

Hence, we have

$$\widehat{\sigma}_q^2 \xrightarrow{p} \sigma^2\{1 + q\delta^2\tau(1 - \tau)\}. \tag{4}$$

Combining the results in (3) and (4), we obtain the desired result:

$$M(q) \xrightarrow{p} \frac{q\delta^2\tau(1 - \tau)}{1 + \delta^2\tau(1 - \tau)}$$

where  $\delta = \sigma^{-1}|\mu_1 - \mu_2|$ .

We first note that the probability limit of  $M(q)$  is positive and depends on three parameters: the standardized break size  $\delta$ , the break time  $\tau$ , and the holding period  $q$ . It is obvious from the expression that when there is no break (either  $\delta = 0$  or  $\tau = 0, 1$ ), the probability limit collapses to zero, which corresponds to the standard result. An immediate consequence of Theorem 1 is that the standardized variance ratio statistic  $z(q)$  diverges to infinity at the rate of  $T^{1/2}$ ; that is,  $T^{-1/2}z(q) = O_p(1)$ . Thus, a routine application of the statistic  $z(q)$  based on critical values from  $N(0, 1)$  is likely to result in spurious rejections of the random walk hypothesis even though the null is true. As noted before, the severity of spurious rejections would depend on three parameters:  $\delta$ ,  $\tau$ , and  $q$ . A closer examination of the probability limit of  $M(q)$  predicts that the phenomenon of spurious rejections would be more pronounced when (i) the holding period  $q$  increases, (ii) the standardized break size  $\delta$  becomes larger,

and (iii) the structural break occurs in the middle of the sample size ( $\tau = 0.5$ ). These predictions are confirmed in Figure 1 in which we graph the probability limit against  $\delta$  and  $\tau$  while fixing the holding period  $q$  at two. The graph clearly shows that the probability limit is an increasing function of  $\delta$  and is maximised when  $\tau = 0.5$ .

### 3. Monte Carlo Simulations

In the previous section, the phenomenon of spurious rejections has been demonstrated in large samples. Naturally, it may be interesting to investigate whether the same phenomenon can occur in finite samples and, if it can, how serious the size distortion might be in such circumstances. We generate data through (1) and (2). The error terms  $\varepsilon_t$  are drawn from  $N(0, 1)$ . We normalise  $\mu_1$  at zero and use various values of  $\mu_2 : \mu_2 = 0.1, 0.2, 0.3, 0.4$ . Since  $\sigma = 1$  and  $\mu_1 = 0$ , the standardized break size  $\delta$  is now equal to  $\mu_2$ . In the simulations, we set  $T = 500, 1000$  and  $q = 4, 8$ . The number of replications in all experiments is 1000. In Figure 2, for the specified values of  $T$  and  $q$ , we plot the rejection probability of the test  $z(q)$  at the 5% significance level against the break fraction  $\tau$  ranging from 0 to 1. Figure 2(i) displays the results when  $T = 500$  and  $q = 4$ , which indicates that the size distortion is fairly mild for smaller values of  $\mu_2$ . However, when the break fraction is around 0.5 and the standardized break size becomes larger, the rejection probability can reach up to 30%. As we increase the holding period  $q$  from 4 to 8 [Figure 2(ii)], and as we increase the sample size  $T$  from 500 to 1000 [Figure 2(iii) and Figure 2(iv)], the size distortion becomes more pronounced. Obviously, the size distortion disappears as either  $\tau \rightarrow 0$  or  $\tau \rightarrow 1$  in all figures.

In Figure 2, we have investigated only the case in which there is an increase in drift; i.e.  $\mu_1 < \mu_2$ . We have simulated the opposite cases (a decrease in drift) by setting  $\mu_2 = -0.1, -0.2, -0.3, -0.4$ . The results, as expected from Theorem 1, are entirely symmetric and the plots are identical to the ones in Figure 2. Hence, we do not report them.

Lo and MacKinlay [10] also proposed a variance ratio test that is robust to general forms of heteroscedasticity using the heteroscedasticity-consistent results of [22, 23, 24]. The heteroscedasticity-robust variance ratio test statistic, denoted by  $z^*(q)$ , is given by

$$z^*(q) = \hat{V}^{-1/2}M(q), \tag{5}$$

where

$$\hat{V} = \sum_{j=1}^{q-1} \left[ \frac{2(q-j)}{q} \right]^2 \hat{\delta}(j),$$

$$\hat{\delta}(j) = \frac{\sum_{k=j+1}^T (r_k - \hat{\mu})^2 (r_{k-j} - \hat{\mu})^2}{\left[ \sum_{k=1}^T (r_k - \hat{\mu})^2 \right]^2}.$$

Under the null hypothesis that returns are heteroscedastic but uncorrelated, Lo and MacKinlay showed that  $z^*(q)$  is asymptotically distributed as  $N(0, 1)$ . We have repeated the same experiments as shown in Figure 2, but replacing  $z(q)$  with  $z^*(q)$ . We have found that the

results are virtually the same; that is, the same phenomenon of spurious rejections occurs and the magnitude of spurious rejections is identical for both  $z(q)$  and  $z^*(q)$ . For this reason, the results are not reported in this paper. Hence, using the heteroscedasticity-robust variance ratio test cannot provide any protection against the size distortion problem when there is a break in drift.

#### 4. Modified Variance Ratio Tests

In many applications, it is usually assumed that structural break points are known a priori. For example, see [9] and [6]. Assuming that the break fraction  $\tau$  is known, the spurious rejection problem described in the previous sections can be fixed by slightly modifying the detrending procedure in the original variance ratio tests. Instead of demeaning the series  $r_t$  using the whole sample, we demean  $r_t$  in each of the subsamples. This can be done by a simple regression. We consider the following regression in which we regress  $r_t$  on a constant and a dummy variable  $d_t$  defined as  $d_t = 1[\tau T < t]$

$$r_t = \hat{\beta}_0 + \hat{\beta}_1 d_t + \tilde{r}_t,$$

where  $\tilde{r}_t$  is the residual from the above LS regression. Our modified variance ratio statistic denoted  $z_m(q)$  is now calculated based on  $\tilde{r}_t$  as follows<sup>†</sup>:

$$z_m(q) = T^{1/2} \left( \frac{2(2q - 1)(q - 1)}{3q} \right)^{-1/2} M_m(q) \tag{6}$$

where

$$\begin{aligned} M_m(q) &= \frac{\hat{\sigma}_{qm}^2}{\hat{\sigma}_m^2} - 1, \\ \hat{\sigma}_m^2 &= \frac{1}{T - 1} \sum_{t=1}^T \tilde{r}_t^2, \\ \hat{\sigma}_{qm}^2 &= \frac{T}{q(T - q + 1)(T - q)} \sum_{t=q}^T \left( \sum_{i=0}^{q-1} \tilde{r}_{t-i} \right)^2. \end{aligned}$$

Using the fact that  $\hat{\beta}_0 \xrightarrow{p} \mu_1$  and  $\hat{\beta}_1 \xrightarrow{p} \mu_2 - \mu_1$ , it is straightforward to show that  $z_m(q)$  converges in distribution to  $N(0, 1)$ . Moreover, the same modification can be used to make the heteroscedasticity-robust variance ratio test in (5) robust. We have repeated the same Monte Carlo experiments as shown in Figure 2, but replacing  $z(q)$  with the new test  $z_m(q)$ . The results are displayed in Figure 3 for the specified values of  $T$  and  $q$ , and it is clearly demonstrated that the spurious rejection phenomenon has now disappeared.

We note that our regression-based procedure can easily be extended to a situation in which there are multiple breaks as long as the break points are assumed to be known. For example,

<sup>†</sup>The subscript  $m$  indicates that the test is a modified version of the corresponding variance-ratio test.

suppose that there are three break points denoted  $\tau_1, \tau_2$  and  $\tau_3$ . In this case, we run the following regression:

$$r_t = \hat{\beta}_0 + \hat{\beta}_1 d_{1t} + \hat{\beta}_2 d_{2t} + \hat{\beta}_3 d_{3t} + \tilde{r}_t \quad (7)$$

where  $d_{1t} = 1 [\tau_1 T < t \leq \tau_2 T - 1]$ ,  $d_{2t} = 1 [\tau_2 T < t \leq \tau_3 T - 1]$  and  $d_{3t} = 1 [\tau_3 T < t]$ . The correctly modified variance ratio test is now obtained using the new residual  $\tilde{r}_t$  from the above regression (7) in the formula in (6).

## 5. An Empirical Example

The random walk process in (1) and (2) exhibits, under our investigation, a deterministic break in its drift term: for instance, from positive to negative or vice versa. If the drift term changes its sign from positive to negative, then this set-up can be a plausible model for a time series on exchange rates which display a persistent currency appreciation period followed by a long depreciation era. Likewise, the reverse case of changing from negative to positive can correspond to when the exchange rates exhibit the opposite dynamics. In this section, we apply our modified variance ratio testing procedure to revisit the random walk hypothesis for exchange rates.

Since many countries shifted to a floating exchange rate system during the period from 1970 to 1973, our data set starts on January 2, 1974. We use weekly exchange rates for the following four currencies against the U.S. dollar: Canadian dollar (CAN), German mark (DM), Italian lira (ITL), and Swiss franc (SZF). The reasons for using exchange rates from these four countries are that (i) the preliminary application of the standardized heteroscedasticity-robust variance ratio test  $z^*(q)$  indicates that the random walk hypothesis is rejected for exchange rates for these countries and (ii) the exchange rate data from these countries clearly show that there are three distinctive structural breaks as displayed in Figure 4.

Exchange rates for the Canadian dollar and the Swiss franc against the U.S. dollar end on October 15, 2003. Due to the introduction of the euro on January 1, 1999, data only extends to December 30, 1998 for the German mark and the Italian lira. Exchange rates used in our study are the noon buying rates in New York for cable transfers payable in foreign currencies. All exchange rates are measured by unit of foreign currency per U.S. dollar and can be downloaded from the Federal Reserve board's website (<http://www.federalreserve.gov/releases/h10/Hist>). We denote by  $S_t$  Wednesday's exchange rates and the corresponding weekly returns  $r_t$  are calculated through  $r_t = \ln S_t - \ln S_{t-1}$ . If Wednesday's exchange rate is missing due to a holiday, Thursday's exchange rate (or Tuesday's if Thursday's is missing) is used because holidays occur least often on Wednesdays and Thursdays. There are 1555 weekly observations of exchange rates for the Canadian dollar and the Swiss franc and 1304 of exchange rates for the German mark and the Italian lira.

Figure 4 provides time series plots of the weekly exchange rates for the currencies of the four countries against the U.S. dollar during the post-Bretton-Woods system of flexible exchange rates. As clearly indicated in the figures, three structural break points (denoted  $\tau_1, \tau_2$  and  $\tau_3$ ) might exist. Exchange rates for the German mark, Italian lira and Swiss franc have common structural break points: January 1980 ( $\tau_1$ ), March 1985 ( $\tau_2$ ) and December

1987 ( $\tau_3$ ). Exchange rates for the German mark and the Italian lira generally tend to move together from March 1979, when Germany and Italy joined the Exchange Rate Mechanism (ERM) under the European Monetary System, except for the period from September 1992 to November 1996. This later disparity is due to the fact that Italy seceded from the ERM in September 1992 and re-entered in November 1996. Exchange rates for the Swiss franc move very similarly to exchange rates for the German mark. The intermediate target of the Swiss National Bank was an exchange rate of 0.8SZF/DM during the period from 1975 to 1980.

For these three currencies, the first break point of January 1980 indicates the start of their depreciation against the U.S. dollar. The combination of high interest rates and the low inflation rate in the U.S. at that time caused the dollar to appreciate against most other currencies. The real interest rate in the U.S. exhibited a sudden increase in the year 1980, which was associated with a change in the Federal Reserve's monetary control procedure and the Depository Institutions Deregulation Act. The three currencies continued to weaken against the U.S. dollar throughout the early 1980s, and the value of the U.S. dollar reached all-time highs early in 1985.

The second break point of March 1985 indicates the switch from the appreciation era of the U.S. dollar to the depreciation period. This structural break could be related to two events: (i) the Reagan administration's commitment to the floating exchange rate system started to change when Regan and Sprinkel were succeeded at the Treasury by Baker and Darman in January 1985; and (ii) in September 1985, the G-5 countries reached the Plaza Agreement to bring down the value of the U.S. dollar. The dollar's value declined persistently from March 1985 to the end of 1987, mainly due to the large trade deficit in the U.S. and governments' interventions.

The third break point of December 1987 denotes the end of the U.S. dollar's depreciation era, this change being caused by a decrease in the level of the U.S. trade deficit and high U.S. interest rates. The third break point was about 10 months after the Louvre Agreement that the economic policy makers from the G-5 countries and Canada agreed in February 1987, namely that exchange rates should be stabilized around the prevailing levels. After the third break point, the value of the U.S. dollar became stabilized until the end of the sample period in December 1998.

The value of the Canadian dollar in terms of the U.S. dollar exhibits a quite different pattern from that of the other three, European, currencies. The Canadian dollar's exchange rate floated within a narrower band. The value of the Canadian currency appears to have had three structural breaks: February 1986 ( $\tau_1$ ), October 1991 ( $\tau_2$ ) and January 2002 ( $\tau_3$ ), as indicated in Figure 4(i). During the second half of the 1970s, the value of the Canadian dollar against the U.S. dollar fell, even though the U.S. dollar was depreciating against other major currencies because of political uncertainty and a substantial current account deficit in Canada. Unlike other, European, currencies, the Canadian dollar's exchange rate does not show a structural break in the early 1980s as noted by [9]. Although the major currencies began to appreciate against the U.S. dollar after the Plaza Agreement, the Canadian dollar continued to depreciate against the U.S. dollar due to concerns about weakening Canadian economic prospects. In February 1986, four months after the Plaza Agreement, the Canadian dollar appreciated in value, following a concerted strategy of aggressive intervention in the



exchange market, higher interest rates and the announcement of large borrowings by the Canadian government [17]. After Canada agreed to the Louvre Agreement, the Canadian dollar continued to strengthen until October 1991 due to the interventions and a tightening of monetary policy. In November 1991, the Canadian dollar began to depreciate, largely because of low interest rates, large fiscal and current account deficits in Canada, and softening commodity prices. After almost a decade of this depreciation, the Canadian dollar started to appreciate again in January 2002 caused by the Canadian trade surplus and the general depreciation of the U.S. dollar against all major floating currencies, itself most likely caused by concern over U.S. fiscal and current account deficits.

Table 1: Descriptive Statistics for Returns on Weekly Exchange Rates<sup>‡</sup>

	CAN	DM	SZF	ITL
full sample period	Jan. 2, '74 - Oct. 15, '03	Jan. 2, '74 - Dec. 30, '98	Jan. 2, '74 - Oct. 15, '03	Jan. 2, '74 - Dec. 30, '98
$T$	1554	1303	1554	1303
$\hat{\mu}$	0.00018	-0.00038	-0.00059	0.00076
$s.e.(\hat{\mu})$	(0.00017)	(0.00044)	(0.00045)	(0.00043)
subperiod 1	Jan. 2, '74 - Feb. 5, '86	Jan. 2, '74 - Jan. 9, '80	Jan. 2, '74 - Jan. 9, '78	Jan. 2, '74 - Jan. 2, '80
$T_1$	631	314	314	313
$\hat{\mu}_1$	0.00059	-0.00152	-0.00239	0.00084
$s.e.(\hat{\mu}_1)$	(0.00024)	(0.00068)	(0.00088)	(0.00071)
subperiod 2	Feb. 12, '86 - Oct. 30, '91	Jan. 16, '80 - Mar. 6, '85	Jan. 16, '78 - Mar. 6, '85	Jan. 9, '80 - Mar. 13, '85
$T_2$	299	269	269	271
$\hat{\mu}_2$	-0.00081	0.00253	0.00227	0.00356
$s.e.(\hat{\mu}_2)$	(0.00030)	(0.00098)	(0.00108)	(0.00086)
subperiod 3	Nov. 6, '91 - Jan. 23, '02	Mar. 13, '85 - Dec. 30, '87	Mar. 13, '85 - Dec. 30, '87	Mar. 20, '85 - Dec. 30, '87
$T_3$	571	147	147	146
$\hat{\mu}_3$	0.00061	-0.00513	-0.00551	-0.00397
$s.e.(\hat{\mu}_3)$	(0.00027)	(0.00130)	(0.00146)	(0.00113)
subperiod 4	Oct. 16, '02 - Oct. 15, '03	Jan. 6, '88 - Dec. 30, '98	Jan. 6, '88 - Oct. 15, '03	Jan. 6, '88 - Dec. 30, '98
$T_4$	53	573	824	573
$\hat{\mu}_4$	-0.00349	0.00008	0.00003	0.00060
$s.e.(\hat{\mu}_4)$	(0.00152)	(0.00065)	(0.00059)	(0.00068)

After identifying the possible break points, we use a formal testing procedure to check if there is statistical evidence to justify the choice. With three break points, we have four

possible drift terms denoted  $\mu_1, \mu_2, \mu_3$  and  $\mu_4$ . Each term is estimated by

$$\hat{\mu}_i = \frac{1}{[\tau_i T] - [\tau_{i-1} T]} \sum_{t=[\tau_{i-1} T]+1}^{[\tau_i T]} r_t,$$

where  $r_t = \ln S_t - \ln S_{t-1}$ ,  $\tau_0 = 0$  and  $\tau_4 = 1$ . Table 1 presents the estimates of drift terms,  $\hat{\mu}_i$ , their standard errors and the number of observations,  $T$ , for the full sample, for each sub-period, and for each country. The estimates of drift terms  $\hat{\mu}$  based on the full sample are not statistically significant for all countries. We have tested the null hypothesis  $H_0 : \mu_i = \mu_{i+1}$  ( $i = 1, 2, 3$ ) and  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$  for each country using the  $t$ -test and the  $F$ -test based on the HAC standard errors. The nulls are rejected for all sub-periods and for all countries at the 1% significance level. The  $t$ -values and  $F$ -values are reported in Table 2. The estimates of  $\hat{\mu}_i$  are significantly different across the subsamples and the changing sign of  $\hat{\mu}_i$  is entirely consistent with our previous discussion of the appreciation and depreciation periods; that is, the sign of the sample mean is significantly positive (negative) when the U.S. dollar exchange rate persistently appreciates (depreciates), which is equivalent to having a positive (negative) drift term.

Table 2:  $t$ -values and  $F$ -values Based on HAC Standard Errors

$H_0$	CAN	DM	SZF	ITL
$\mu_1 = \mu_2$	3.515	-3.680	-3.593	-2.611
$\mu_2 = \mu_3$	-3.399	4.470	4.115	4.813
$\mu_3 = \mu_4$	2.587	-3.304	-3.256	-3.098
$\mu_1 = \mu_2 = \mu_3 = \mu_4$	8.561	9.801	10.068	9.152

Lo and MacKinlay [11] examined the size and power properties of the standard variance ratio tests for the random walk hypothesis via Monte Carlo simulations and found that the empirical size of the variance ratio test is close to its nominal value under the null hypothesis with independent and identically distributed Gaussian errors as well as with heteroscedastic increments for sample sizes that are greater than 32. But, when the holding period  $q$  increases relative to the sample size, it was also found that the sampling distribution of the standard variance ratio tests deviates significantly from the asymptotic standard normal distribution. Fong et al. [6] also conducted a simulation study, suggesting a reasonable range of  $q$  for a given value of  $T$ . Taking into account these results in previous research, we take a conservative approach by using  $q = 3, 4, 8$  and  $16$ , given that we have 1554 return observations for CAN and SZF and 1303 return observations for DM and ITL.

First, we apply the  $z(q)$  test to the four exchange rates at 10% and 5% significance levels, assuming that there is no heteroscedasticity in the error term. The testing results are displayed in Table 3. The evidence from the data points strongly against the random walk hypothesis; the null is rejected for  $q = 3, 4$  in CAN, for  $q = 3, 4, 16$  in SZF, and for all values of  $q$  in both DM and ITL. Next, we apply our modified version  $z_m(q)$ , calculated through the auxiliary LS regression in (7), for which the exact dates for  $\tau_1, \tau_2$  and  $\tau_3$  have already been specified

Table 3: Variance Ratio Test Results Using  $z(q)$  and  $z_m(q)$ <sup>§</sup>

		$q$			
		3	4	8	16
CAN	$z(q)$	2.574 <sup>a</sup>	1.874 <sup>b</sup>	0.979	0.307
	$z_m(q)$	1.849 <sup>b</sup>	1.0034	-0.244	-1.399
DM	$z(q)$	2.132 <sup>a</sup>	2.032 <sup>a</sup>	2.067 <sup>a</sup>	2.278 <sup>a</sup>
	$z_m(q)$	1.166	0.886	0.469	0.164
SZF	$z(q)$	1.801 <sup>b</sup>	1.834 <sup>b</sup>	2.383 <sup>a</sup>	2.561 <sup>a</sup>
	$z_m(q)$	0.991	0.877	1.075	0.893
ITL	$z(q)$	1.443	1.846 <sup>b</sup>	2.560 <sup>a</sup>	3.055 <sup>a</sup>
	$z_m(q)$	0.530	0.775	1.055	1.002

above. The results are also in Table 3, which shows that (i) all the modified test statistics become much smaller than the unmodified ones, and (ii) we fail to reject the random walk hypothesis in all series but CAN. Even in that case, we reject the null at the 10% significance level, only when  $q = 3$ .

The strong evidence against the random walk hypothesis by the  $z(q)$  test might have been caused by the possible presence of heteroscedasticity in the considered exchange rates. Hence, we also apply the heteroscedasticity-consistent variance ratio test statistic  $z^*(q)$ , the results of which are in Table 4. Table 4 indicates that heteroscedasticity must have played a role because we now have a much smaller number of rejections of the random walk hypothesis, when compared to the  $z(q)$  test; the null is rejected for  $q = 3$  in CAN, for  $q = 8, 16$  in SZF, for  $q = 8, 16$  in ITL, and for all values of  $q$  in DM. The evidence has been much weakened, but we still reject the null for all countries. Finally, we apply our modified  $z_m^*(q)$  test taking into account both heteroscedasticity and structural breaks in drift. Table 4 shows that the random walk hypothesis is not rejected in all cases.

Table 4: Variance Ratio Test Results Using  $z^*(q)$  and  $z_m^*(q)$ <sup>¶</sup>

		$q$			
		3	4	8	16
CAN	$z^*(q)$	1.961 <sup>a</sup>	1.447	0.780	0.253
	$z_m^*(q)$	1.423	0.783	-0.197	-1.164
DM	$z^*(q)$	1.894 <sup>b</sup>	1.763 <sup>b</sup>	1.773 <sup>b</sup>	1.997 <sup>a</sup>
	$z_m^*(q)$	1.037	0.766	0.400	0.143
SZF	$z^*(q)$	1.632	1.642	2.107 <sup>a</sup>	2.228 <sup>a</sup>
	$z_m^*(q)$	0.895	0.783	0.948	0.794
ITL	$z^*(q)$	1.150	1.433	1.949 <sup>b</sup>	2.411 <sup>a</sup>
	$z_m^*(q)$	0.420	0.598	0.797	0.786

## 6. Summary

We have demonstrated that, if a break in drift is not properly taken into account, then the routine application of the variance ratio test can result in a potentially large size distortion rejecting the random walk hypothesis 100% of the time asymptotically even though the process is actually a random walk. Based on a simple regression approach, we have proposed a modification of the test such that the size of the test is correctly controlled. When the standard variance ratio tests and our modified tests are applied to exchange rates for CAN, DM, ITL and SZF, the test results clearly show that the rejection of the random walk hypothesis by the standard variance ratio tests may have been caused by ignoring the presence of structural breaks.

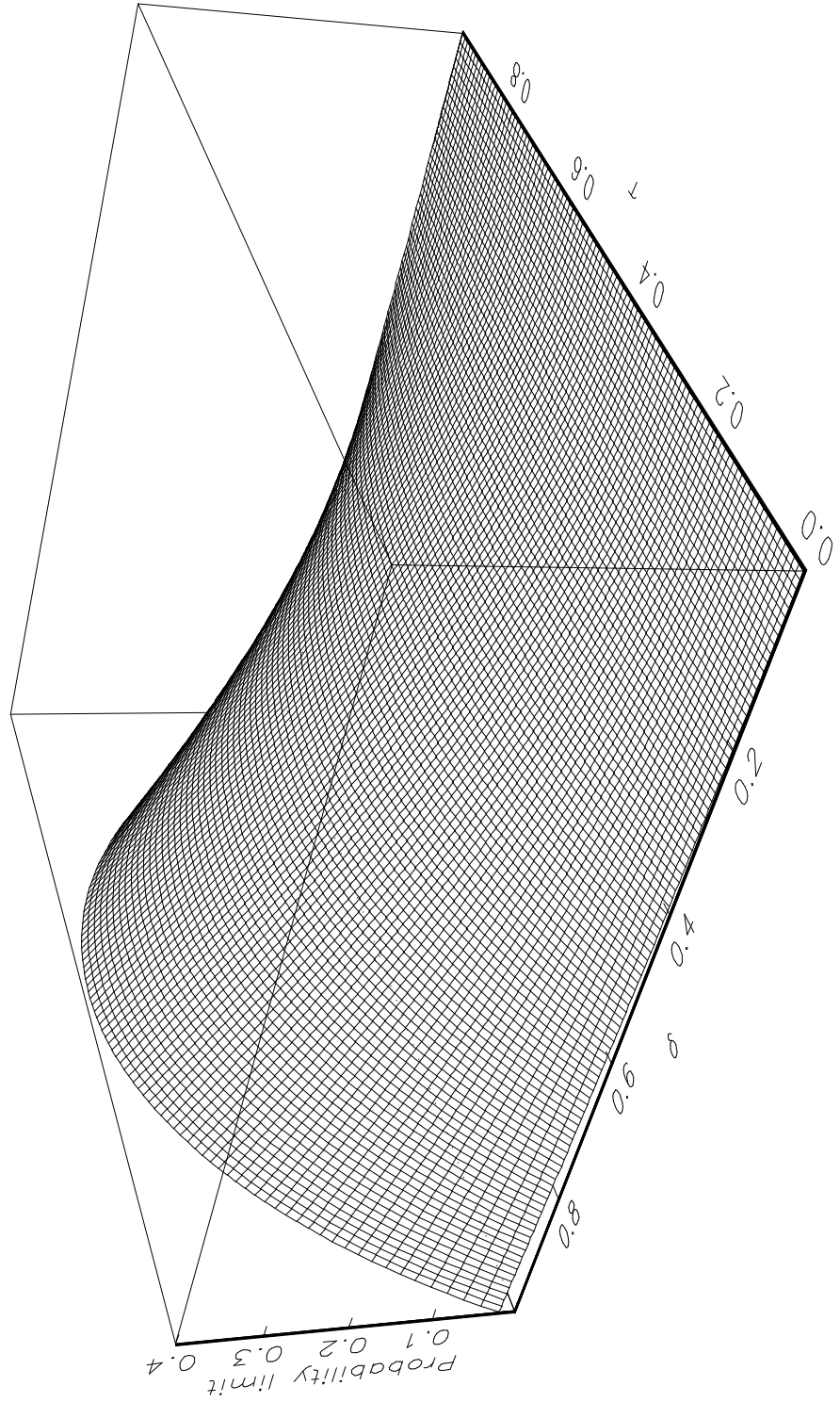
## References

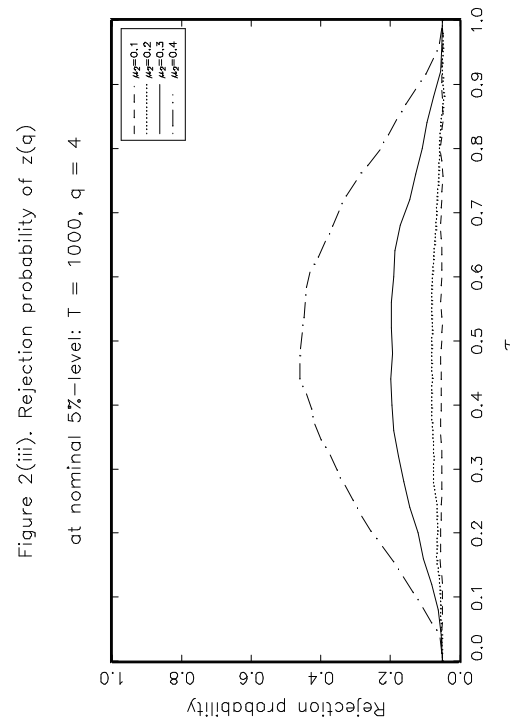
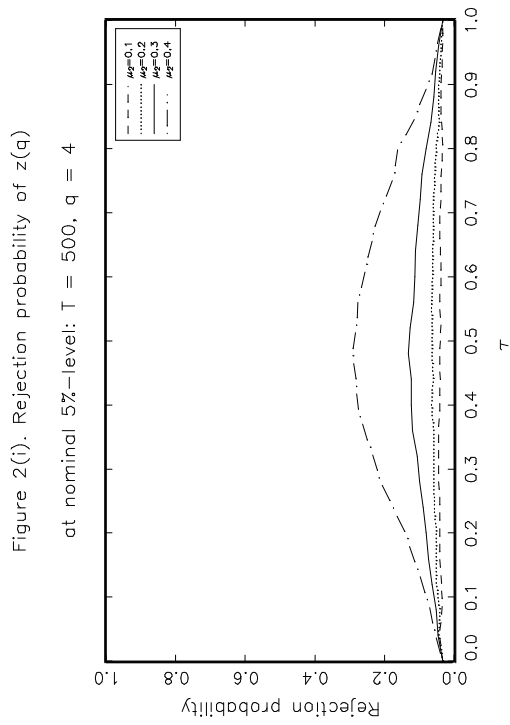
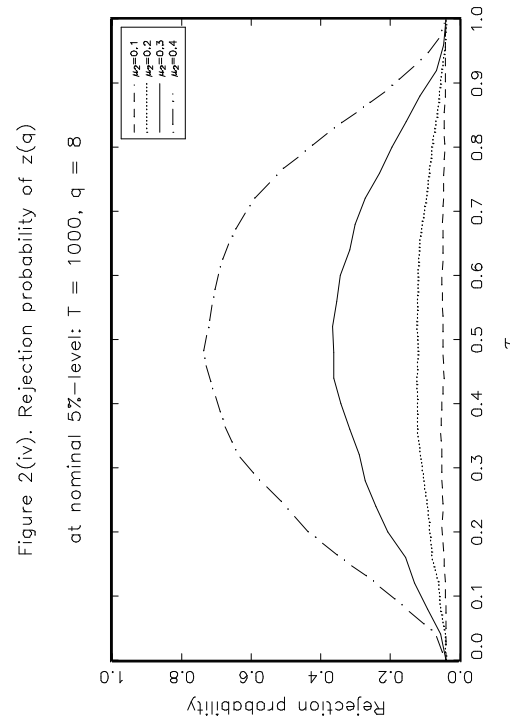
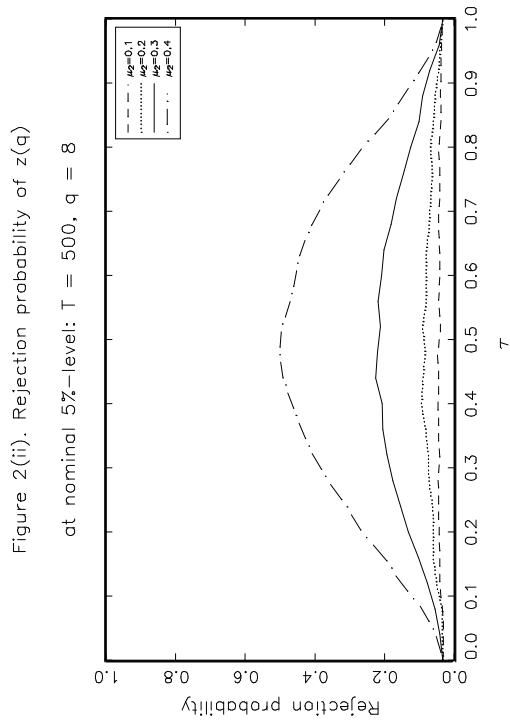
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## Appendix

Figure 1. Probability limit of  $M_r(q)$ .





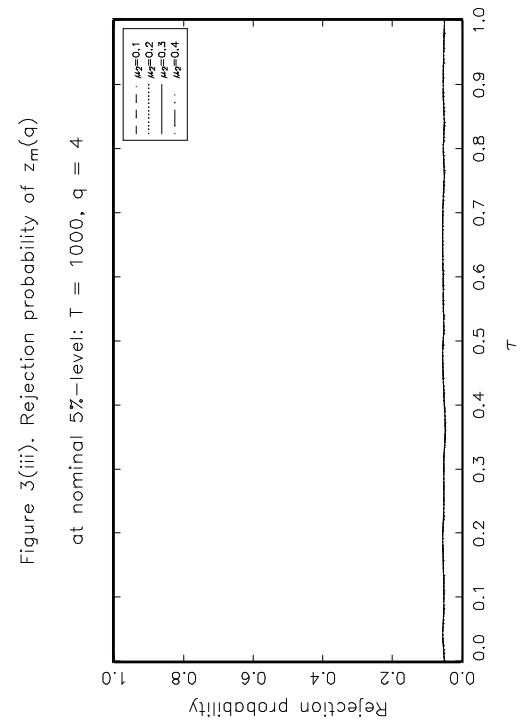
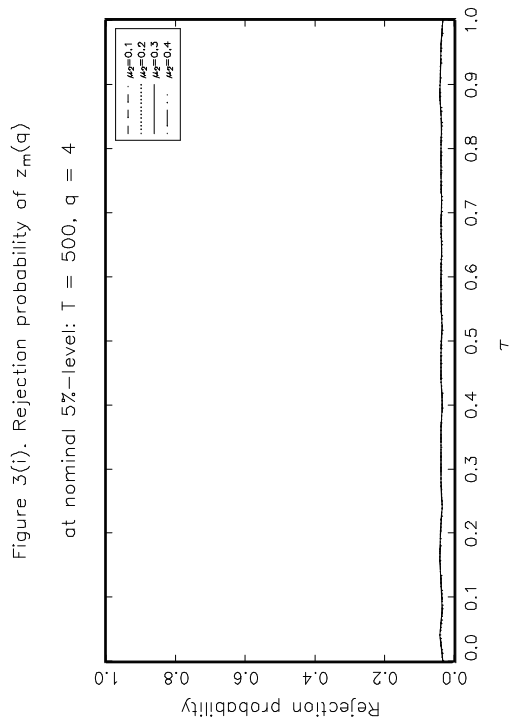
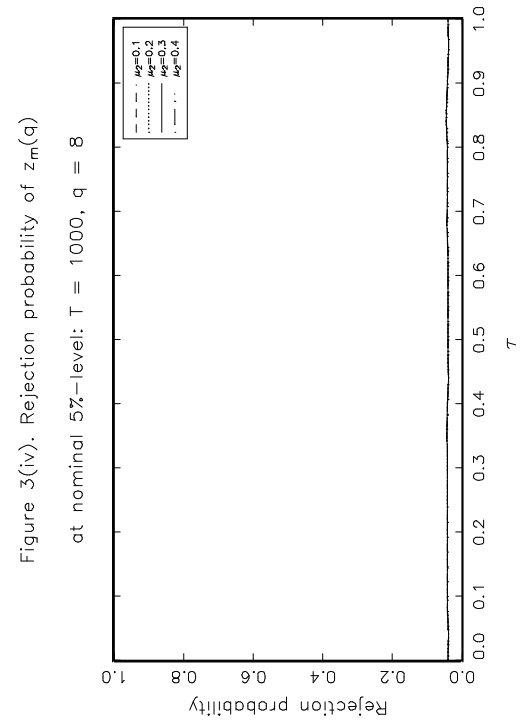
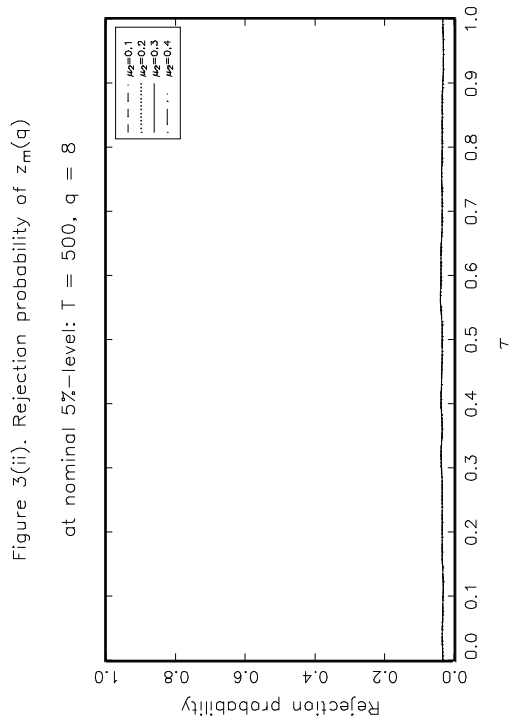




Figure 4(ii). DM/US\$

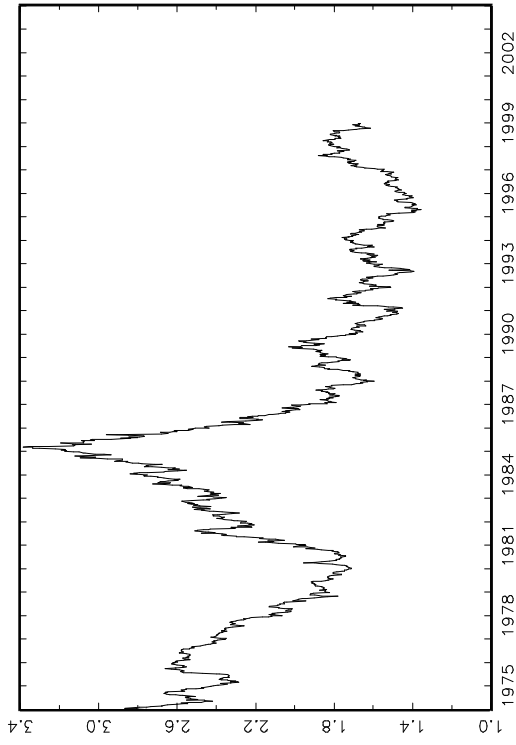


Figure 4(iv). ITL/US\$

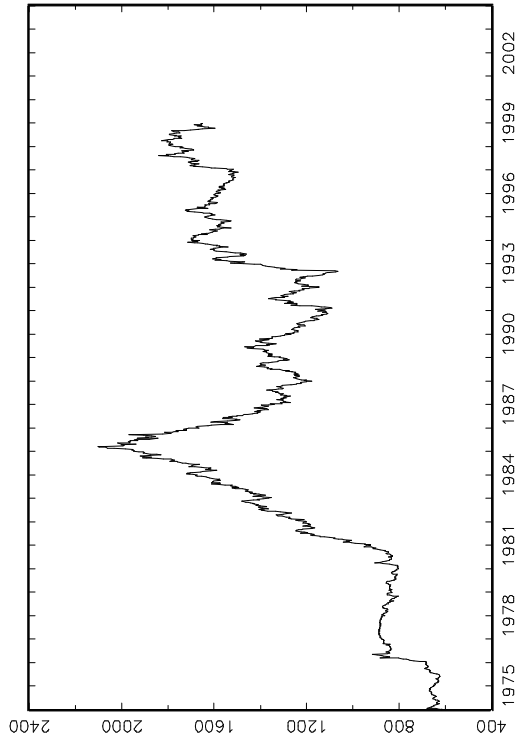


Figure 4(i). CAN/US\$

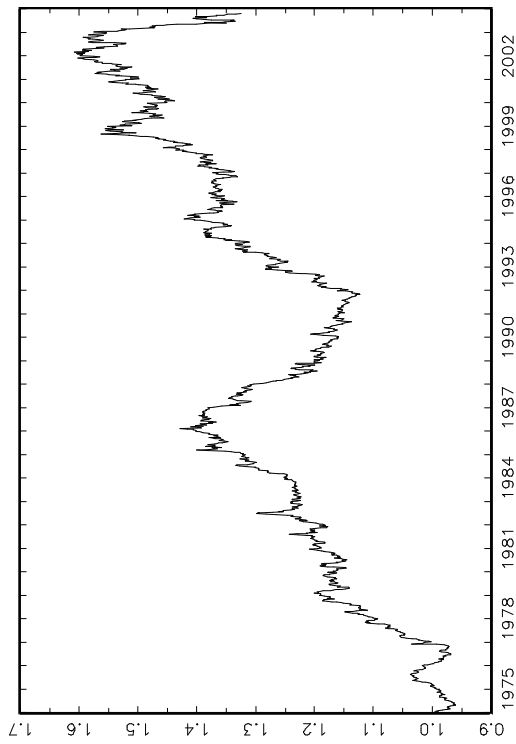


Figure 4(iii). SZF/US\$

