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On $I_{s^\star g}$-Continuous Functions in Ideal Topological Spaces

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Abstract. By using $I_{s^\star g}$-closed sets due to Khan and Hamza [5], we introduce the notion of $I_{s^\star g}$-continuous functions in ideal topological spaces. We obtain several properties of $I_{s^\star g}$-continuity and the relationship between this function and other related functions.

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1. Introduction

Khan and Hamza [5] introduced and investigated the notion of $I_{s^\star g}$-closed sets in ideal topological spaces as a generalization of I_g-closed sets due to Dontchev et al. [2]. In this paper, by using $I_{s^\star g}$-closed sets we introduce $I_{s^\star g}$-continuous functions, strongly $I_{s^\star g}$-continuous functions and weakly $I_{s^\star g}$-continuous functions. It turns out that weak $I_{s^\star g}$-continuity is weaker than weak I-continuity defined by Ackgoz et al. [1] . We obtain several properties of $I_{s^\star g}$-continuity and the relationship between this function and other related functions.

2. Preliminaries

Let (X,τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X,τ) , $cl(A)$ and $Int(A)$ denote the closure and interior of A in (X,τ), respectively. An ideal I on a set X is a non-empty collection of subsets of X which satisfies the following properties:

(1) $A \in I$ and $B \subset A$ implies $B \in I$,

(2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

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An ideal topological space is a topological space (X,τ) with an ideal I on X and is denoted by (X,τ,I). For a subset $A\subset X$, $A^\star(I,\tau)=\{x\in X : A\cap U\notin I \text{ for every } U\in\tau(x)\}$, where, $\tau(x)=\{U\in\tau : x\in U\}$, is called the local function of A with respect to I and τ [4, 6]. We simply write $A^\star$ or $A^\star_X$ instead of $A^\star(I,\tau)$ and $B^\star_A$ for $B^\star(I_A,\tau_A)$ in case there is no chance for confusion. For every ideal topological space (X,τ,I), there exists a topology $\tau^\star(I)$, finer than τ, generated by the base $\beta(I,\tau)=\{U-J : U\in\tau \text{ and } J\in I\}$. It is known in [4] that $\beta(I,\tau)$ is not necessarily a topology. When there is no ambiguity, $\tau^\star(I)$ is denoted by $\tau^\star$. Recall that A is said to be $()^\star$-dense in itself (resp. $\tau^\star$-closed, $()^\star$-perfect) if $A\subset A^\star$ (resp. $A^\star\subset A$, $A=A^\star$). For a subset A of X, $cl^\star(A)$ and $Int^\star(A)$ will, respectively, denote the closure and interior of A in $(X,\tau^\star)$. A subset A of X is said to be semi-open [7] if there exists an open set U in X such that $U\subset A\subset cl(U)$. The complement of a semi-open set is said to be semi-closed. A subset A is said to be semi-regular if A is semi-open and semi-closed. A subset A of X is said to be generalized closed [8] (briefly, g-closed) if $cl(A)\subset U$ whenever $A\subset U$ and U is open in X. The complement of a g-closed set is said to be g-open. A space X is called a $T_{1/2}$-space [3] if every g-closed set in X is closed. Recall that if (X,τ,I) is an ideal topological space and A is a subset of X, then (A,τ_A,I_A) is an ideal topological space, where τ_A is the relative topology on A and $I_A=\{A\cap J : J\in I\}$.

3. $I_{s^\star g}$-Closed Sets

The notion of $I_{s^\star g}$-closed sets was defined by Khan and Hamza [5]. In this section we will obtain further properties of $I_{s^\star g}$-closed sets in ideal topological spaces.

Definition 1. *A subset A of a space (X,τ,I) is said to be $I_{s^\star g}$-closed [5] if $A^\star\subset U$ whenever $A\subset U$ and U is semi-open in X. The complement of an $I_{s^\star g}$-closed set is said to be $I_{s^\star g}$-open, equivalently if $F\subset Int^\star(A)$ whenever $F\subset A$ for every semi-closed set F in X.*

Lemma 1. *Every open set is $I_{s^\star g}$-open.*

Lemma 2 ([Lemma 2.7, 2])**.** *Let (X,τ,I) be an ideal topological space and $B\subset A\subset X$. Then $B^\star(I_A,\tau_A)=B^\star(I,\tau)\cap A$.*

Lemma 3. *If U is open and A is $I_{s^\star g}$-open, then $U\cap A$ is $I_{s^\star g}$-open.*

Proof. We prove that $X-(U\cap A)$ is $I_{s^\star g}$-closed. Let $X-(U\cap A)\subset G$ where G is semi-open in X. This implies $(X-U)\cup(X-A)\subset G$. Since $(X-A)\subset G$ and $(X-A)$ is $I_{s^\star g}$-closed in X, therefore $(X-A)^\star\subset G$. Moreover $X-U$ is closed and contained in G, therefore, $(X-U)^\star\subset cl(X-U)\subset G$. Hence $(X-(U\cap A))^\star=((X-U)\cup(X-A))^\star=(X-U)^\star\cup(X-A)^\star\subset G$. This proves that $U\cap A$ is $I_{s^\star g}$-open.

Theorem 1. *Let (X,τ,I) be an ideal topological space and $B\subset A\subset X$. If B is an $I_{s^\star g}$-closed set relative to A, where A is open and $I_{s^\star g}$-closed in X, then B is $I_{s^\star g}$-closed in X.*

Proof. Let $B\subset G$, where G is semi-open in X. Then $B\subset A\cap G$ and $A\cap G$ is semi-open in X and hence in A. Therefore $B^\star_A\subset A\cap G$. It follows from Lemma 2 that $A\cap B^\star_X\subset A\cap G$ or

$A \subset G \cup (X - B_X^\star)$. By Theorem 2.3 of [4], $B_X^\star$ is closed in X and $G \cup (X - B_X^\star)$ is semi-open in X. Since A is $I_{s^\star g}$-closed in X, $A_X^\star \subset G \cup (X - B_X^\star)$ and hence $B^\star = B^\star \cap A^\star \subset B^\star \cap [G \cup (X - B_X^\star)] \subset G$. Therefore, we obtain $B_X^\star \subset G$. This proves that B is $I_{s^\star g}$-closed in X.

Theorem 2. *Let A be a semi-open set in a space (X, τ, I) and $B \subset A \subset X$. If B is $I_{s^\star g}$-closed in X, then B is $I_{s^\star g}$-closed relative to A.*

Proof. Let $B \subset U$ where U is semi-open in A. Then there exists a semi-open set V in X such that $U = A \cap V$. Thus $B \subset A \cap V$. Now $B \subset V$ implies that $B_X^\star \subset V$. It follows that $A \cap B_X^\star \subset A \cap V$. By Lemma 2, $B_A^\star \subset A \cap V = U$. This proves that B is a $I_{s^\star g}$-closed relative to A.

Corollary 1. *Let $B \subset A \subset X$ and A be open and $I_{s^\star g}$-closed in (X, τ, I). Then B is $I_{s^\star g}$-closed relative to A if and only if B is $I_{s^\star g}$-closed in X.*

Theorem 3. *If B is a subset of a space (X, τ, I) such that $A \subset B \subset A^\star$ and A is $I_{s^\star g}$-closed in X, then B is also $I_{s^\star g}$-closed in X.*

Proof. Let G be a semi-open set in X containing B, then $A \subset G$. Since A is $I_{s^\star g}$-closed, therefore $A^\star \subset G$ and hence $B^\star \subset (A^\star)^\star \subset A^\star \subset G$. This implies that B is $I_{s^\star g}$-closed in X.

Theorem 4. *Let $B \subset A \subset X$ and suppose that B is $I_{s^\star g}$-open in X and A is a semi-regular set in X. Then B is $I_{s^\star g}$-open relative to A.*

Proof. We prove that $A - B$ is $I_{s^\star g}$-closed relative to A. Let $U \in SO(A)$ such that $(A - B) \subset U$. Now $(A - B) \subset (X - B) \subset U \cup (X - A)$, where $U \cup (X - A) \in SO(X)$ because $A \in SR(X)$. Since $X - B$ is $I_{s^\star g}$-closed in X, therefore $(X - B)_X^\star \subset U \cup (X - A)$ or $(X - B)_X^\star \cap A \subset (U \cup (X - A)) \cap A \subset U$. By Lemma 2, $(A - B)_A^\star = (A - B)_X^\star \cap A \subset (X - B)_X^\star \cap A \subset U$ and hence $(A - B)_A^\star \subset U$. This proves that B is $I_{s^\star g}$-open relative to A.

Theorem 5. *Let $B \subset A \subset X$. B is $I_{s^\star g}$-open in A and A is open in X then B is $I_{s^\star g}$-open in X.*

Proof. Let F be a semi-closed subset of B in X. Since A is open, therefore $F \in SC(A)$. Since B is $I_{s^\star g}$-open in A, therefore $F \subset Int_A^\star(B) = A \cap Int_X^\star(B) \subset Int_X^\star(B)$. This proves that B is $I_{s^\star g}$-open in X.

4. $I_{s^\star}g$-Continuous Functions

Definition 2. *A function $f : (X, \tau, I) \to (Y, \Omega, J)$ is said to be weakly I-continuous [1] if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists an open set U containing x such that $f(U) \subset cl^\star(V)$.*

Definition 3. *A function $f : (X, \tau, I) \to (Y, \Omega)$ is said to be $I_{s^\star g}$-continuous if for every $U \in \Omega$, $f^{-1}(U)$ is $I_{s^\star g}$-open in (X, τ_X, I).*

Remark 1. *Every continuous function is $I_{s^{\star}g}$-continuous and the converse need not be true as seen from Example 2 (below).*

Definition 4. *A function $f : (X, \tau) \rightarrow (Y, \Omega, J)$ is said to be strongly $I_{s^{\star}g}$-continuous if for every $I_{s^{\star}g}$-open set U in Y, $f^{-1}(U)$ is open in X.*

Remark 2. *Every strongly $I_{s^{\star}g}$-continuous function is continuous but the converse is not true in general.*

Example 1. *Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Let $Y = \{a, b, c, d\}$ with $\Omega = \{\phi, Y, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $J = \{\phi, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \Omega, J)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = a$ and $f(d) = d$. Then f is continuous. Let $U = \{a, c\}$ then U is $I_{s^{\star}g}$-open in Y but $f^{-1}(U) = \{a, c\}$ is not open in X. Hence f is not strongly $I_{s^{\star}g}$-continuous.*

Definition 5. *A function $f : (X, \tau, I) \rightarrow (Y, \Omega, J)$ is said to be weakly $I_{s^{\star}g}$-continuous if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists an $I_{s^{\star}g}$-open set U containing x such that $f(U) \subset cl^{\star}(V)$.*

Remark 3. *(1) Every weakly I-continuous function is weakly $I_{s^{\star}g}$-continuous but the converse is not true in general.*

(2) Every $I_{s^{\star}g}$-continuous function is weakly $I_{s^{\star}g}$-continuous.

By the above definitions, for a function $f : (X, \tau, I) \rightarrow (Y, \Omega, J)$ we obtain the following implications:

strong $I_{s^{\star}g}$-continuity	$\Rightarrow$	continuity	$\Rightarrow$	$I_{s^{\star}g}$-continuity
		$\Downarrow$		$\Downarrow$
		Weak I-continuity	$\Rightarrow$	weak $I_{s^{\star}g}$-continuity

Remark 4. *$I_{s^{\star}g}$-continuity and weak I-continuity are independent of each other.*

Example 2. *Let $X = Y = \{a, b, c, d\}$ and $\tau = \Omega = \{\phi, X, \{a, b\}\}$ with $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Define $f : (X, \tau, I) \rightarrow (Y, \Omega, I)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$ and $f(d) = b$. Then f is $I_{s^{\star}g}$-continuous but not weak I-continuous. Since for $c \in X$, $f(c) = b$ and an open set $V = \{a, b\}$ containing $f(c)$, the only open set containing c is $U = X$ and $f(U) \nsubseteq cl^{\star}(V) = \{a, b\}$.*

Example 3. *Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, b\}\}$ and $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\Omega = \{\phi, \{1, 2\}, Y\}$ and $J = \{\phi, \{3\}, \{4\}, \{3, 4\}\}$. Define $f : (X, \tau, I) \rightarrow (Y, \Omega, I)$ by $f(a) = 1$, $f(b) = 3$, $f(c) = 2$ and $f(d) = 4$. f is weak I-continuous but not $I_{s^{\star}g}$-continuous. Since $V = \{1, 2\}$ is open in Y but $f^{-1}(V) = \{a, c\}$ is not $I_{s^{\star}g}$-open in X.*

Theorem 6. *Let $f : (X, \tau, I) \rightarrow (Y, \Omega)$ be a function. Then, the following statements are equivalent:*

(1) f is $I_{s^{\star}g}$-continuous.

(2) The inverse image of each closed set in Y is $I_{s^{\star}g}$-closed in X.

(3) The inverse image of each open set in Y is $I_{s^\star g}$-open in X.

Definition 6. *An ideal topological space (X,τ,I) is said to be T-dense if every subset of X is $\star$-dense in itself.*

Definition 7. *Let N be a subset of a space (X,τ,I) and $x\in X$. Then N is called an $I_{s^\star g}$-open neighborhood of x if there exists an $I_{s^\star g}$-open set U containing x such that $U\subset N$.*

Theorem 7. *Let (X,τ,I) be T-dense. Then, for a function $f:(X,\tau,I)\to(Y,\Omega)$ the following statements are equivalent:*

(1) f is $I_{s^\star g}$-continuous.

(2) For each $x\in X$ and each open set V in Y with $f(x)\in V$, there exists an $I_{s^\star g}$-open set U containing x such that $f(U)\subset V$.

(3) For each $x\in X$ and each open set V in Y with $f(x)\in V$, $f^{-1}(V)$ is an $I_{s^\star g}$-open neighborhood of x.

Proof. (1) ⇒ (2) Let $x\in X$ and let V be an open set in Y such that $f(x)\in V$. Since f is $I_{s^\star g}$-continuous, $f^{-1}(V)$ is $I_{s^\star g}$-open in X. By putting $U=f^{-1}(V)$, we have $x\in U$ and $f(U)\subset V$.

(2) ⇒ (3) Let V be an open set in Y and let $f(x)\in V$. Then by (2), there exists an $I_{s^\star g}$-open set U containing x such that $f(U)\subset V$. So $x\in U\subset f^{-1}(V)$. Hence $f^{-1}(V)$ is an $I_{s^\star g}$-open neighbourhood of x.

(3) ⇒ (1) Let V be an open set in Y and let $f(x)\in V$. Then by (3), $f^{-1}(V)$ is an $I_{s^\star g}$-neighborhood of x. Thus for each $x\in f^{-1}(V)$, there exists an $I_{s^\star g}$-open set U_x containing x such that $x\in U_x\subset f^{-1}(V)$. Hence $f^{-1}(V)=\cup_{x\in f^{-1}(V)}U_x$ and so by Theorem 2.12 [5], $f^{-1}(V)$ is $I_{s^\star g}$-open in X.

Theorem 8. *A function $f:(X,\tau)\to(Y,\Omega,J)$ is strongly $I_{s^\star g}$-continuous if and only if the inverse image of every $I_{s^\star g}$-closed set in Y is closed in X.*

Theorem 9. *(1) Let $f:(X,\tau)\to(Y,\Omega,J)$ be strongly $I_{s^\star g}$-continuous and $h:(Y,\Omega,J)\to(Z,\sigma)$ be $I_{s^\star g}$-continuous, then $h\circ f$ is continuous.*

(2) Let $f:(X,\tau,I)\to(Y,\Omega)$ be $I_{s^\star g}$-continues and $g:(Y,\Omega)\to(Z,\sigma)$ be continuous, then $g\circ f:(X,\tau,I)\to(Z,\sigma)$ is $I_{s^\star g}$-continuous.

Theorem 10. *Let $f:(X,\tau,I)\to(Y,\Omega)$ be $I_{s^\star g}$-continuous and $U\in RO(X)$. Then the restriction $f\mid U:(U,\tau_U,I_U)\to(Y,\Omega)$ is $I_{s^\star g}$-continuous.*

Proof. Let V be any open set of (Y,τ_Y). Since f is $I_{s^\star g}$-continuous, $f^{-1}(V)$ is $I_{s^\star g}$-open in X. By Theorem 2.14 of [5], $f^{-1}(V)\cap U$ is $I_{s^\star g}$-open in X. Thus by Theorem 4 $(f\mid U)^{-1}(V)=f^{-1}(V)\cap U$ is $I_{s^\star g}$-open in U because U is regular-open in X. This proves that $f\mid U:(U,\tau\mid U,I\mid U)\to(Y,\tau_Y)$ is $I_{s^\star g}$-continuous.

Theorem 11. *Let $f : (X,\tau,I) \to (Y,\Omega,J)$ be a function and $\{U_\alpha : \alpha \in \nabla\}$ be an open cover of a T-dense space X. If the restriction $f \mid U_\alpha$ is $I_{s^\star g}$-continuous for each $\alpha \in \nabla$, then f is $I_{s^\star g}$-continuous.*

Proof. Suppose F is an arbitrary open set in (Y,Ω,J). Then for each $\alpha \in \nabla$, we have $(f \mid U_\alpha)^{-1}(V) = f^{-1}(V) \cap U_\alpha$. Because $f \mid U_\alpha$ is $I_{s^\star g}$-continuous, therefore,$f^{-1}(V) \cap U_\alpha$ is $I_{s^\star g}$-open in X for each $\alpha \in \nabla$. Since for each $\alpha \in \nabla$, U_α is open in X, by Theorem 5, $f^{-1}(V) \cap U_\alpha$ is $I_{s^\star g}$-open in X. Now since X is T-dense, by [Theorem 2.12 5], $\cup_{\alpha\in\nabla} f^{-1}(V) \cap U_\alpha = f^{-1}(V)$ is $I_{s^\star g}$-open in X. This implies f is $I_{s^\star g}$-continuous.

Theorem 12. *If (X,τ,I) is a T-dense space and $f : (X,\tau,I) \to (Y,\Omega)$ is $I_{s^\star g}$-continuous, then graph function $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is $I_{s^\star g}$-continuous.*

Proof. Let $x \in X$ and W be any open set in $X \times Y$ containing $g(x) = (x, f(x))$. Then there exists a basic open set $U \times V$ such that $g(x) \subset U \times V \subset W$. Since f is $I_{s^\star g}$-continuous, there exits an $I_{s^\star g}$-open set U_1 in X containing x such that $f(U_1) \subset V$. By Lemma 3, $U_1 \cap U$ is $I_{s^\star g}$-open in X and we have $x \in U_1 \cap U \subset U$ and $g(U_1 \cap U) \subset U \times V \subset W$. Since X is T-dense, therefore by Theorem 7, g is $I_{s^\star g}$-continuous.

Theorem 13. *A function $f : (X,\tau,I) \to (Y,\Omega)$ is $I_{s^\star g}$-continuous if the graph function $g : X \to X \times Y$ is $I_{s^\star g}$-continuous.*

Proof. Let V be an open set in Y containing $f(x)$. Then $X \times V$ is an open set in $X \times Y$ and by the $I_{s^\star g}$-continuity of g, there exists an $I_{s^\star g}$-open set U in X containing x such that $g(U) \subset X \times V$. Therefore, we obtain $f(U) \subset V$. This shows that f is $I_{s^\star g}$-continuous.

Theorem 14. *Let $\{X_\alpha : \alpha \in \nabla\}$ be any family of topological spaces. If $f : (X,\tau,I) \to \Pi_{\alpha\in\nabla} X_\alpha$ is an $I_{s^\star g}$-continuous function, then $P_\alpha \circ f : X \to X_\alpha$ is $I_{s^\star g}$-continuous for each $\alpha \in \nabla$, where P_α is the projection of ΠX_α onto X_α.*

Proof. We will consider a fixed $\alpha_0 \in \nabla$. Let G_{α_0} be an open set of X_{α_0}. Then $(P_{\alpha_0})^{-1}(G_{\alpha_0})$ is open in ΠX_α. Since f is $I_{s^\star g}$-continuous, $f^{-1}((P_{\alpha_0})^{-1}(G_{\alpha_0})) = (P_{\alpha_0} \circ f)^{-1}(G_{\alpha_0})$ is $I_{s^\star g}$-open in X. Thus $P_\alpha \circ f$ is $I_{s^\star g}$-continuous.

Corollary 2. *For any bijective function $f : (X,\tau) \to (Y,\Omega,J)$, the following are equivalent:*

(1) $f^{-1} : (Y,\Omega,J) \to (X,\tau)$ is $I_{s^\star g}$-continuous.

(2) $f(U)$ is $I_{s^\star g}$-open in Y for every open set U in X.

(3) $f(U)$ is $I_{s^\star g}$-closed in Y for every closed set U in X.

Proof. It is trivial.

Definition 8. *An ideal topological space (X,τ,I) is an RI-space [1], if for each $x \in X$ and each open neighbourhood V of x, there exists an open neighbourhood U of x such that $x \in U \subset cl^\star(U) \subset V$.*

Theorem 15. *Let (Y,Ω,J) be an RI-space and (X,τ,I) be a T-dense space. Then $f:(X,\tau,I)\rightarrow(Y,\Omega,J)$ is weak $I_{s^{\star}g}$-continuous if and only if f is $I_{s^{\star}g}$-continuous.*

Proof. The sufficiency is clear.

Necessity. Let $x\in X$ and V be an open set of Y containing $f(x)$. Since Y is an RI-space, there exists an open set W of Y such that $f(x)\in W\subset cl^{\star}(W)\subset V$. Since f is weakly $I_{s^{\star}g}$-continuous, there exists an $I_{s^{\star}g}$-open set U such that $x\in U$ and $f(U)\subset cl^{\star}(W)$. Hence we obtain that $f(U)\subset cl^{\star}(W)\subset V$. By Theorem 8, f is $I_{s^{\star}g}$-continuous.

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