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On $\delta\alpha$, δp and δs-Irresolute Functions

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Abstract. In this paper is to introduce and investigate new classes of various irresolute functions and obtain some of their properties in topological spaces.

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1. Introduction

Recall the concepts of α-open [26] (resp. semi-open [15], preopen [18], β-open [11]) sets and semi α-irresolute [2] (resp. semi α-preirresolute [3]) functions in topological spaces.

The main purpose of this paper is to define and study the notions of new classes of functions, namely $\delta\alpha$-irresolute, δp-irresolute and δs-irresolute functions, and to give some properties of these functions in topological spaces.

2. Preliminaries

Throughout this paper, spaces always mean topological spaces and $f : X \to Y$ denotes a single valued function of a space (X, τ) into a space (Y, σ). Let S be a subset of a space (X, τ). The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively.

Here we recall the following known definitions and properties.

Definition 1. *A subset S of a space (X, τ) is said to be α-open [26] (resp. semi-open [15], preopen [18], β-open [11]) if $S \subset Int(Cl(Int(S)))$ (resp. $S \subset Cl(Int(S))$, $S \subset Int(Cl(S))$, $S \subset Cl(Int(Cl(S)))$).*

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A point $x \in X$ is called the δ-cluster point of A if $A \cap Int(Cl(U)) \neq \varnothing$ for every open set U of X containing x. The set of all δ-cluster points of A is called the δ-cluster of A, denoted by $Cl_\delta(A)$. A subset A of X is called δ-closed [27] if $A = Cl_\delta(A)$. The complement of a δ-closed set is called δ-open [27]. A subset A of X is said to be a δ-semiopen [23] if there exists a δ-open set U of X such that $U \subset A \subset Cl(U)$. The complement of a δ-semiopen set is called δ-semiclosed set. A point $x \in X$ is called the δ-semicluster point of A if $A \cap U \neq \varnothing$ for every δ-semiopen set U of X containing x. The set of all δ-semicluster points of A is called the δ-semiclosure [23] of A, denoted by $\delta Cl_s(A)$. The family of all α-open (resp. semi-open, preopen, β-open, δ-open, δ-semiopen) sets in a space (X, τ) is denoted by $\tau^\alpha = \alpha(X)$ (resp. $SO(X)$, $PO(X)$, $\beta O(X)$, $\delta O(X)$, $\delta SO(X)$). It is shown in [26] that τ^α is a topology for X. Moreover, $\tau \subset \tau^\alpha = PO(X) \cap SO(X) \subset \beta O(X)$.

The complement of an α-open (resp. preopen, semi-open) set is said to be α-closed [17] (resp. preclosed [18], semi-closed [8]). The intersection of all α-closed (resp. preclosed, semi-closed) sets in (X, τ) containing a subset A is called the α-closure [17] (resp. preclosure [10], semi-closure [8]) of A, denoted by $\alpha Cl(A)$ (resp. $pCl(A)$, $sCl(A)$).

The union of all α-open (resp. preopen, semi-open, δ-open) sets of X contained in A is called the α-interior [1] (resp. preinterior [19], semi-interior [8], δ-interior [27]) of A and is denoted by $\alpha lnt(A)$ (resp. $plnt(A)$, $sInt(A)$, $Int_\delta(A)$).

A subset S of a space (X, τ) is δ-semiopen [23] (resp. δ-semiclosed) if $S \subset Cl(Int_\delta(S))$ (resp. $Int(Cl_\delta(S)) \subset S$).

Lemma 1 (Park et al. [23])**.** *The intersection (resp. union) of arbitrary collection of δ-semiclosed (resp. δ-semiopen) sets in (X,τ) is δ-semiclosed (resp. δ-semiopen). And $A \subset X$ is δ-semiclosed if and only if $A = \delta Cl_s(A)$.*

Lemma 2 ([7, 10, 22, 14])**.** *Let $\{X_\lambda : \lambda \in \Lambda\}$ be any family of topological spaces and U_{λ_i} be a nonempty subset of X_{λ_i} for each $i = 1, 2, \ldots, n$. Then $U = \Pi_{\lambda \neq \lambda_i} X_\lambda \times \Pi_{i=1}^n U_{\lambda_i}$ is a nonempty α-open [7] (resp. preopen [10], semi-open [22], δ-semiopen [14]) subset of ΠX_λ if and only if U_{λ_i} is α-open (resp. preopen, semi-open, δ-semiopen) in X_{λ_i} for each $i = 1, 2, \ldots, n$.*

Lemma 3. *Let A and B be subsets of a space (X, τ). Then we have*

(1) If $A \epsilon \delta SO(X)$ and $B \epsilon \delta O(X)$, then $A \cap B \epsilon \delta SO(B)$ [14].

(2) If $A \epsilon \delta SO(B)$ and $B \epsilon \delta O(X)$, then $A \epsilon \delta SO(X)$ [6].

Definition 2. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:*

- *semi α-irresolute [2] if $f^{-1}(V)$ is semi-open set in X for every α-open subset V of Y.*
- *semi α-preirresolute [3] if $f^{-1}(V)$ is semi-open set in X for every preopen subset V of Y.*
- *(δ,β)-irresolute [6] if $f^{-1}(V)$ is δ-semiopen set in X for every β-open subset V of Y.*
- *δ-semi-continuous [25] if $f^{-1}(V)$ is δ-semiopen set in X for every open subset V of Y.*
- *α-irresolute [17] if $f^{-1}(V)$ is α-open set in X for every α-open subset V of Y.*

- *preirresolute [24] if $f^{-1}(V)$ is preopen set in X for every preopen subset V of Y.*
- *irresolute [9] if $f^{-1}(V)$ is semi-open set in X for every semi-open subset V of Y.*

3. $\delta\alpha$, δp and δs-Irresolute Functions

Definition 3. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be $\delta\alpha$-irresolute, δp-irresolute and δs-irresolute if $f^{-1}(V)$ is δ-semiopen set in X for every α-open (resp. preopen, semi-open) subset V of Y.*

From the definitions, we have the following relationships:

$$\begin{array}{ccccc}
(\delta,\beta)-\text{irresoluteness} & \rightarrow & \delta p-\text{irresoluteness} & \rightarrow & \text{semi}-\alpha-\text{preirresoluteness} \\
\downarrow & & \downarrow & & \downarrow \\
\delta s-\text{irresoluteness} & \rightarrow & \delta\alpha-\text{irresoluteness} & \rightarrow & \text{semi}-\alpha-\text{irresoluteness}
\end{array}$$

However the converses of the above implications are not true in general by the following examples.

Example 1. *Let $X=\{a,b,c\}$ with topologies $\tau=\{X,\emptyset,\{a,b\}\}$ and $\sigma=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}$. Let a function $f:(X,\tau)\to(X,\sigma)$ be defined by $f(a)=f(b)=a$ and $f(c)=c$. Then f is semi α-preirresolute and hence semi α-irresolute but it is neither $\delta\alpha$-irresolute, δs-irresolute nor δp-irresolute.*

Example 2. *Let $X=\{a,b,c,d\}$ with topologies $\tau=\{X,\emptyset,\{a,b,c\}\}$ and $\sigma=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. Let a function $f:(X,\tau)\to(X,\sigma)$ be defined by $f(a)=f(b)=f(c)=b$ and $f(d)=c$. Then f is δs-irresolute and hence $\delta\alpha$-irresolute but it is not semi α-preirresolute.*

Theorem 1. *For a function $f:(X,\tau)\to(Y,\sigma)$, the following are equivalent:*

(a) f is $\delta\alpha$-irresolute;

(b) $f:(X,\tau)\to(Y,\sigma^{\alpha})$ is δ-semi-continuous;

(c) For each $x\in X$ and each α-open set V of Y containing $f(x)$, there exists a δ-semiopen set U of X containing x such that $f(U)\subset V$;

(d) $f^{-1}(V)\subset Cl(Int_{\delta}(f^{-1}(V)))$ for every α-open set V of Y;

(e) $f^{-1}(F)$ is δ-semiclosed in X for every α-closed set F of Y;

(f) $Int(Cl_{\delta}(f^{-1}(B)))\subset f^{-1}(\alpha Cl(B))$ for every subset B of Y;

(g) $f(Int(Cl_{\delta}(A)))\subset \alpha Cl(f(A))$ for every subset A of X.

Proof. **(a)** $\Rightarrow$ **(b)**. Let $x \in X$ and V be any α-open set of Y containing $f(x)$. By Definition 3, $f^{-1}(V) \epsilon \delta SO(X)$ containing x and hence $f : (X,\tau) \to (Y,\sigma^{\alpha})$ is δ-semi-continuous.

(b) $\Rightarrow$ **(c)**. Let $x \in X$ and V be any α-open set of Y containing $f(x)$. Set $U = f^{-1}(V)$, then by (b), U is a δ-semiopen set of X containing x and $f(U) \subset V$.

(c) $\Rightarrow$ **(d)**. Let V be any α-open subset of Y and $x \in f^{-1}(V)$. By (c), there exists a δ-semiopen set U of X containing x such that $f(U) \subset V$. Therefore, we obtain
$x \in U \subset Cl(Int_{\delta}(U)) \subset Cl(Int_{\delta}(f^{-1}(V)))$ and hence $f^{-1}(V) \subset Cl(Int_{\delta}(f^{-1}(V)))$.

(d) $\Rightarrow$ **(e)**. Let F be any α-closed subset of Y. Set $V = Y - F$, then V is α-open in Y. By (d), we have $f^{-1}(V) \subset Cl(Int_{\delta}(f^{-1}(V)))$ and hence $f^{-1}(F) = X - (f^{-1}(Y-F)) = X - f^{-1}(V)$ is δ-semiclosed in X.

(e) $\Rightarrow$ **(f)**. Let B be any subset of Y. Since $\alpha Cl(B)$ is α-closed in Y, $f^{-1}(\alpha Cl(B))$ is δ-semiclosed in X and hence $Int(Cl_{\delta}(f^{-1}(\alpha Cl(B)))) \subset f^{-1}(\alpha Cl(B))$. Thus we have $Int(Cl_{\delta}(f^{-1}(B))) \subset f^{-1}(\alpha Cl(B))$.

(f) $\Rightarrow$ **(g)**. Let A be any subset of X. By (f), we obtain
$Int(Cl_{\delta}(A)) \subset Int(Cl_{\delta}(f^{-1}(f(A)))) \subset f^{-1}(\alpha Cl(f(A)))$ and hence $f(Int(Cl_{\delta}(A))) \subset \alpha Cl(f(A))$.

(g) $\Rightarrow$ **(a)**. Let V be any α-open subset of Y. Since $f^{-1}(Y - V) = X - f^{-1}(V)$ is a subset of X and by (g), we obtain

$$f(Int(Cl_{\delta}(f^{-1}(Y-V)))) \subset \alpha Cl(f(f^{-1}(Y-V))) \subset \alpha Cl(Y-V) = Y - \alpha Int(V) = Y - V$$

and hence

$$\begin{aligned} X - Cl(Int_{\delta}(f^{-1}(V))) =& Int(Cl_{\delta}(X - f^{-1}(V))) \\ =& Int(Cl_{\delta}(f^{-1}(Y-V))) \subset f^{-1}(f(Int(Cl_{\delta}(f^{-1}(Y-V))))) \subset f^{-1}(Y-V) \\ =& X - f^{-1}(V). \end{aligned}$$

Therefore, we have $f^{-1}(V) \subset Cl(Int_{\delta}(f^{-1}(V)))$ and hence $f^{-1}(V)$ is δ-semiopen in X. Thus the function f is $\delta\alpha$-irresolute.

Now, the proofs of the following two theorems are similar to Theorem 1 and are thus omitted.

Theorem 2. *The following are equivalent for a function $f : (X,\tau) \to (Y,\sigma)$:*

(a) f is δp-irresolute;

(b) For each $x \in X$ and each preopen set V of Y containing $f(x)$, there exists a δ-semiopen set U of X containing x such that $f(U) \subset V$;

(c) $f^{-1}(V) \subset Cl(Int_{\delta}(f^{-1}(V)))$ for every preopen set V of Y;

(d) $f^{-1}(F)$ is δ-semiclosed in X for every preclosed set F of Y;

(e) $Int(Cl_{\delta}(f^{-1}(B))) \subset f^{-1}(pCl(B))$ for every subset B of Y;

(f) $f(Int(Cl_{\delta}(A))) \subset pCl(f(A))$ for every subset A of X.

Theorem 3. *For a function $f:(X,\tau)\to(Y,\sigma)$, the following are equivalent:*

(a) f is δs-irresolute;

(b) For each $x\in X$ and each semi-open set V of Y containing $f(x)$, there exists a δ-semiopen set U of X containing x such that $f(U)\subset V$;

(c) $f^{-1}(V)\subset Cl(Int_\delta(f^{-1}(V)))$ for every semi-open set V of Y;

(d) $f^{-1}(F)$ is δ-semiclosed in X for every semi-closed set F of Y;

(e) $Int(Cl_\delta(f^{-1}(B)))\subset f^{-1}(sCl(B))$ for every subset B of Y;

(f) $f(Int(Cl_\delta(A)))\subset sCl(f(A))$ for every subset A of X.

The proofs of the other parts of the following theorems follow by a similar way and are thus omitted.

Theorem 4. *Let $f:X\to Y$ be a function and $g:X\to X\times Y$ the graph function, given by $g(x)=(x,f(x))$ for every $x\in X$. Then f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) if g is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).*

Proof. Let $x\in X$ and V be any α-open (resp. preopen,semi-open) set of Y containing $f(x)$. Then, by Lemma 2, The set $X\times V$ is α-open (resp. preopen, semi-open) in $X\times Y$ containing $g(x)$. Since g is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), there exists a δ-semiopen set U of X containing x such that $g(U)\subset X\times V$ and hence $f(U)\subset V$. Thus f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).

Theorem 5. *If a function $f:X\to\Pi Y_\lambda$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $P_\lambda o f:X\to Y_\lambda$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) for each $\lambda\in\Lambda$, where P_λ is the projection of ΠY_λ onto Y_λ.*

Proof. Let V_λ be any α-open (resp. preopen, semi-open) set of Y_λ. Since P_λ is continuous and open, it is α-irresolute [20, Theorem 3.2] (resp. preirresolute [20, Theorem 3.4], irresolute [9, Theorem 1.2]) and hence $P_\lambda^{-1}(V_\lambda)$ is α-open (resp. preopen, semi-open) in ΠY_λ. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $f^{-1}(P_\lambda^{-1}(V_\lambda))=(P_\lambda o f)^{-1}(V_\lambda)$ is δ-semiopen in X. Hence $P_\lambda o f$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) for each $\lambda\in\Lambda$.

Theorem 6. *If the product function $f:\Pi X_\lambda\to\Pi Y_\lambda$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $f_\lambda:X_\lambda\to Y_\lambda$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) for each $\lambda\in\Lambda$.*

Proof. Let $\lambda_0\in\Lambda$ be an arbitrary fixed index and V_{λ_0} be any α-open (resp. preopen, semi-open) set of Y_{λ_0}. Then $\Pi Y_\lambda\times V_{\lambda_0}$ is α-open (resp. preopen, semi-open) in ΠY_λ by Lemma 2, where $\lambda_0\neq\lambda\in\Lambda$. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $f^{-1}(\Pi Y_\lambda\times V_{\lambda_0})=\Pi X_\lambda\times f_{\lambda_0}^{-1}(V_{\lambda_0})$ is δ-semiopen in ΠX_λ and hence, by Lemma 2, $f_{\lambda_0}^{-1}(V_{\lambda_0})$ is δ-semiopen in X_{λ_0}. This implies that f_{λ_0} is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).

Theorem 7. *If $f : (X,\tau) \to (Y,\sigma)$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) and A is a δ-open subset of X, then the restriction $f_{/A} : A \to Y$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).*

Proof. Let V be any α-open (resp. preopen, semi-open) set of Y. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $f^{-1}(V)$ is δ-semiopen in X. Since A is δ-open in X, $(f_{/A})^{-1}(V) = A \cap f^{-1}(V)$ is δ-semiopen in A by the condition (1) of Lemma 3. Hence $f_{/A}$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).

Theorem 8. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function and $\{A_\lambda : \lambda \in \Lambda\}$ be a cover of X by δ-open sets of (X,τ). Then f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) if $f_{/A_\lambda} : A_\lambda \to Y$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) for each $\lambda \in \Lambda$.*

Proof. Let V be any α-open (resp. preopen, semi-open) set of Y. Since $f_{/A_\lambda}$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $(f_{/A_\lambda})^{-1}(V) = f^{-1}(V) \cap A_\lambda$ is δ-semiopen in A_λ. Since A_λ is δ-open in X, by the condition (2) of Lemma 3, $(f_{/A_\lambda})^{-1}(V)$ is δ-semiopen in X for $\lambda \in \Lambda$. Therefore $f^{-1}(V) = X \cap f^{-1}(V) = \cup\{A_\lambda \cap f^{-1}(V) : \lambda \in \Lambda\} = \cup\{(f_{/A_\lambda})^{-1}(V) : \lambda \in \Lambda\}$ is δ-semiopen in X because the union of δ-semiopen sets is a δ-semiopen set by Lemma 1. Hence f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).

Theorem 9. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition $gof : X \to Z$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) if f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) and g is α-irresolute (resp. preirresolute, irresolute).*

Proof. Let W be any α-open (resp. preopen, semi-open) subset of Z. Since g is α-irresolute (resp. preirresolute, irresolute), $g^{-1}(W)$ is α-open (resp. preopen, semi-open) in Y. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), then $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is δ-semiopen in X and hence gof is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute).

We recall that a space (X,τ) is said to be submaximal [4] if every dense subset of X is open in X and extremally disconnected [26] if the closure of each open subset of X is open in X. The following theorem follows from the fact that if (X,τ) is a submaximal and extremally disconnected space, then $\tau = \tau^\alpha = SO(X) = PO(X) = \beta O(X)$ [12, 21].

Theorem 10. *Let (Y,σ) be a submaximal and extremally disconnected space and let $f : (X,\tau) \to (Y,\sigma)$ be a function. Then we have*

$$\delta\alpha\text{-irresoluteness} \Leftrightarrow \delta p\text{-irresoluteness} \Leftrightarrow \delta s - \text{irresoluteness} \Leftrightarrow (\delta,\beta)\text{-irresoluteness}.$$

Recall that a topological space (X,τ) is called $\alpha - T_2$ [17] (resp. pre-T_2 [13], semi-T_2 [16], δ-semi-T_2 [5] if for any distinct pair of points x and y in X, there exist $U \epsilon \alpha(X,x)$ and $V \epsilon \alpha(X,y)$ (resp. $U \epsilon PO(X,x)$ and $V \epsilon PO(X,y)$, $U \epsilon SO(X,x)$ and $V \epsilon SO(X,y)$, $U \epsilon \delta SO(X,x)$ and $V \epsilon \delta SO(X,y)$) such that $U \cap V = \emptyset$.

Theorem 11. *If $f : (X,\tau) \to (Y,\sigma)$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) injection and Y is $\alpha - T_2$ (resp. pre-T_2, semi-T_2), then X is δ-semi-T_2.*

Proof. Let x and y be distinct points of X. Then $f(x) \neq f(y)$. Since Y is $\alpha - T_2$ (resp. pre-T_2, semi-T_2), there exist disjoint α-open (resp. preopen, semi-open) sets V and W containing $f(x)$ and $f(y)$, respectively. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), there exist δ-semiopen sets U and H containing x and y, respectively, such that $f(U) \subset V$ and $f(H) \subset W$. It follows that $U \cap H = \varnothing$. This shows that X is δ-semi-T_2.

Lemma 4 (Lee et. al. [14]). *If A_i is a δ-semiopen set of $X_i (i = 1, 2)$, then $A_1 \times A_2$ is δ-semiopen in $X_1 \times X_2$.*

Theorem 12. *If $f : (X, \tau) \to (Y, \sigma)$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) and Y is $\alpha - T_2$ (resp. pre-T_2, semi-T_2), then the set $E = \{(x, y) : f(x) = f(y)\}$ is δ-semiclosed in $X \times X$.*

Proof. Suppose that $(x, y) \notin E$. Then $f(x) \neq f(y)$. Since Y is $\alpha - T_2$ (resp. pre-T_2, semi-T_2), there exist $V \epsilon \alpha(Y, f(x))$ and $W \epsilon \alpha(Y, f(y))$ (resp. $V \epsilon PO(Y, f(x))$ and $W \epsilon PO(Y, f(y))$, $V \epsilon SO(Y, f(x))$ and $W \epsilon SO(Y, f(y))$) such that $V \cap W = \varnothing$. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), there exist δ-semiopen sets U and H containing x and y, respectively, such that $f(U) \subset V$ and $f(H) \subset W$. Set $G = U \times H$. By Lemma 4, $(x, y) \in G \in \delta SO(X \times X)$ and $G \cap E = \varnothing$. This means that $\delta Cl_s(E) \subset E$ and hence the set E is δ-semiclosed in $X \times X$.

Definition 4. *For a function $f : X \to Y$, the graph $G(f) = \{(x, f(x)) : x \in X\}$ is called $\delta\alpha$-closed (resp. δp-closed, δs-closed) if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \epsilon \delta SO(X, x)$ and $V \epsilon \alpha(Y, y)$ (resp. $V \epsilon PO(Y, y)$, $V \epsilon SO(Y, y)$) such that $(U \times V) \cap G(f) = \varnothing$.*

Theorem 13. *If $f : X \to Y$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) and Y is $\alpha - T_2$ (resp. pre-T_2, semi-T_2), then $G(f)$ is $\delta\alpha$-closed (resp. δp-closed, δs-closed) in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. This implies that $f(x) \neq y$. Since Y is $\alpha - T_2$ (resp. pre-T_2, semi-T_2), there exist disjoint α-open (resp. preopen, semi-open) sets V and W in Y containing $f(x)$ and y, respectively. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), there exists a δ-semiopen set U of X containing x such that $f(U) \subset V$. Therefore $f(U) \cap W = \varnothing$ and hence $(U \times W) \cap G(f) = \varnothing$. Thus $G(f)$ is $\delta\alpha$-closed (resp. δp-closed, δs-closed) in $X \times Y$.

Theorem 14. *If $f : X \to Y$ is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute) injection with a $\delta\alpha$-closed (resp. δp-closed, δs-closed) graph, then X is δ-semi-T_2.*

Proof. Let x and y be any distinct points of X. Then $f(x) \neq f(y)$ and hence $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is $\delta\alpha$-closed (resp. δp-closed, δs-closed), there exist $U \epsilon \delta SO(X, x)$ and $V \epsilon \alpha(Y, f(y))$ (resp. $V \epsilon PO(Y, f(y))$, $V \epsilon SO(Y, f(y))$) such that $(U \times V) \cap G(f) = \varnothing$ and hence $f(U) \cap V = \varnothing$. Since f is $\delta\alpha$-irresolute (resp. δp-irresolute, δs-irresolute), there exists $G \epsilon \delta SO(X, y)$ such that $f(G) \subset V$. Thus we have $f(U) \cap f(G) = \varnothing$ and hence $U \cap G = \varnothing$. This shows that X is δ-semi-T_2.

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